Toward Application of Sandpile Model in Image Segmentation Based on Extremal Optimization Heuristic

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Abstract: *The sandpile model is a paradigm of self organizing critically (SOC) concept that is inspired by a physics-based intuition for optimization. In this paradigm, physical properties of sandpiles such as avalanche promise improved convergence and lower computations. Extremal Optimization (EO) algorithm is a general-purpose local search heuristic that is based on SOC. Here, application of sandpile model to image segmentation is proposed. In the proposed model, over-segmented images are submitted to the algorithm. Inspired by sandpile model, similar segments then merge, and by using the energy function in Markov random fields (MRF), EO adjusts the labels of pixels. Results indicate that sandpile model can be applicable to image segmentation.*

Keywords: Sandpile model, extremal optimization, segmentation, merging.

1. Introduction

"In nature, highly specialized, complex structures often emerge when their most inefficient elements are selectively driven to extinction" [1]. The dynamics of systems with emergent complexity, can be described by the concept of "self-organized criticality" (SOC) [2-5] that has been proposed by Bak *et al* (1987, 1988).

Self organized criticality explains common characteristics of complex systems, which is formed by self-organization in a long transient period at the border of stability and chaos [2]. In this state, the propagation of the interactions among the components of the system is inversely proportional to its frequency. In fact, Local interactions between many components in an open system can lead to self-organized criticality. Although most state transitions between the components only affect their neighbours, but once in a while entire avalanches of propagating state transitions cause a major reconfiguration of the system. A model that can show this behaviour is called sandpile model [4,6,7] that has an important role in investigating self-organized criticality, which is supposed to represent in some way processes in natural and social systems [4,7].

In sandpile model a pile is gradually built adding an integer height variable on every site representing the number of sand grains that are on top of each other. The height difference between the neighbouring sites is used as a criterion if sand topples from one site to another. After a transient period the sandpile reaches his critical state. This state characterized by the slope of the pile and the minor perturbation (the addition of a single grain) can produce avalanches of all sizes, giving a power law distribution [6,7].

Furthermore, The concept of SOC has been applied successfully in the Bak–Sneppen model of evolution [8, 9]. In this model a species *i* is characterized by a "fitness" value and the "weakest" species and its closest dependent species are successively selected for adaptive changes, getting assigned new (random) fitness values.

One of the methods based on the Bak–Sneppen mechanism, is named *extremal optimization* (EO) [10-14] who is a dynamic optimization procedure free of selection parameters. There is only mutation operator in EO. So, by always performing mutation on the worst species and its neighbours successively, the individual can improve and evolve itself toward the optimal solution generation by generation. For this purpose a suitable representation should be selected which permits each species to be assigned a quality measure (i.e. fitness) [14].

For the first time in this work , sandpile model is proposed to help extremal optimization algorithm for image segmentation.

Image segmentation [15] is one of the important stages in image analysis. It is the process by which an image is segmented into a group of homogeneous segments, where segment is set of pixels [16]. The segmentation result is the labelling of the image pixels that have common property such as brightness, texture, colour, etc.

Previous work on image segmentation based on SOC, used EO and employed the fitness function in markov random field (MRF) model for recovering a "true" image consisting of a few homogeneous segments from a noisy image by labelling individual pixels [17].

The two major MRF based algorithms for image segmentation [7] are the Simulated Annealing (SA) and Iterated Conditional Modes (ICM). Compared to the SA, the ICM provides reasonable segmentation and shows robust behaviour in most of the cases but strongly depends on the initialization phase.

In [16] hierarchical distributed genetic algorithm (HDGA) is used for segmenting noisy and blurred images based on MRF. The algorithm is unsupervised and parallel. Experimental results show that it is effective at segmenting real images.

In [18] τ-EO is used for image segmentation based on Markov Random Fields model. Noisy Images are given to the algorithm and the robustness of the τ-EO algorithm is tested on different images and provided good segmentation. But it fails to enhance the quality of segmentation of real images.

In order to speed up the segmentation process Bak–Sneppen model and MRF are combined in [19] to define a multiresolution image segmentation approach in which a wavelet transform is used to reduce the size of the system. The initial images are still noisy images and it doesn't have good results for real images.

In fact, segmentation of images in few published results [16,18,19] are mainly concerned with removing the effects of noise from images.

This paper is based on a different Model. The algorithm uses oversegmented images as the initial and the method is extended to work both on segment and pixel level. First, the sandpile model is applied to merge similar segments and then EO algorithm is used for adjusting improper labels of pixels.

The rest of the paper is organized as the following: In section 2, the base of sandpile model and EO algorithm is described. Then definitions and concepts of MRF are summarized in section 3. Our proposed model for image segmentation is stated in section 4. In section 5 we discuss about the results of our model. Finally, Section 6 contains the conclusion.

2. Two models of SOC

In this section the two models based on self-organized critically concept is described.

2.1 Sandpile Model

The concept of self-organized criticality evolved from the 'sandpile' model proposed by Bak *et al* (1987, 1988). In this model there is a square grid of boxes and a simulated grain of sand is continuously dropped into a randomly selected box. The number of grains within a square at position (x,y) is described by a variable Z(x,y). In the base model, a box cannot keep more than three grains [16,17].

The algorithm for the sandpile model is shown in Fig. 1.

At initialization the grid is empty and at each time step, a random cell in the grid is selected and its number of grains is increased by one. If the grain of selected cell has exceeded the critical value of three then each of the four neighbouring cells receives one grain, and the number of grains of the cell itself is decreased by four.

If a cell at the boundary of the grid exceeds the critical value then sand grains leave the system. In the beginning of the process, when most cells are nearly empty, there are only a few cells that can propagate grains to neighbouring cells, and thus there are no large avalanches of toppling sand grains [4]. Redistributions can lead to further instabilities and avalanches of grains in which many grains may be lost from the edges of the grid.

```
repeat
  Select a random (x,y)
  Z(x,y)=Z(x,y)+1
  If Z(x,y)==4 then
      Z(x ± 1,y)= Z(x ± 1,y) +1
      Z(x,y ± 1)= Z(x,y ± 1)+1
      Z(x,y)=Z(x,y)-4
      If Z(x ± 1,y)==4 or Z(x,y ± 1)==4
        Update Z recursively
```

Fig. 1: Structure of the sandpile model

2.2 Bak-Sneppen model and Extremal optimization Heuristic

In Bak-Sneppen model, species are placed on the sites of a lattice and each one is assigned a random value between 0 and 1 that shows it's own independent fitness. At each time step, the one species with the smallest fitness is selected and forced to mutate, having its fitness replaced by a new random value between 0 and 1.

But the change in fitness of the weakest species will cause the fitness of all the species at neighbouring lattice sites be replaced with new random numbers as well. After a number of iterations, the system reaches self-organized criticality state. In that state, almost all species have reached a fitness value greater than a certain threshold. Therefore, a little change of one species will result in co-evolutionary chain reactions called avalanches.

Inspired by the bak-sneppen model, Boettcher and Percus proposed the EO algorithm that successively replaces the value of extremely undesirable variables in a sub-optimal solution with new, random ones. In order to avoid getting trapped in local optimum and also for global improvement of results, Boettcher et al. also introduced τ – EO algorithm by adding a single parameter τ to the basic EO. In the τ-EO heuristic, all variables are selected for state-updating indiscriminately (the process is based on the selection against several objectionable variables). Because of this property, τ – EO heuristic is used in this work.

3. Definitions and Notations

In this section, some notations are briefly defined.

An image $S = \{1, .. t, .., MN\}$ specifies the gray levels for all pixels in an M×N lattice where t is called a site . The gray levels belong to the set A= {0,..., 255}. The labelled image is represented by the vector random variable $X = (X_1, .., X_t, .., X_{MN})$, $Xt \in \{1, .., C\}$ where C is the number of categories. The image observed is represented by the MN-vector random variable Y= $(Y1, ..., Y_{MN})$, $Yt \in \{0, .., 255\}$.

A neighbourhood system $N = (Ni \subset S, i \in S)$ is formed by the subsets Ni of S which are neighbours to pixel i that verifies: (1) $i \notin Ni$ and (2) $j \in Ni \Leftrightarrow i \in Nj$.

A clique $c \subset S$ is a set of points which are all neighbours to each other: $\forall\, r, t \in c, r \in Nt$.

The structure of the neighbourhood system determines the MRF order. In a first order the neighbourhood of a site consists of its four nearest neighbours. For a second order the neighbourhood of a site consists of the eight nearest neighbours used in this paper.

Suppose $X = (X_1, .., X_{MN}) \in \Omega$ where Ω is the set of all possible configurations for labels. X is a MRF according to the neighbourhood system N if:

1. $\forall\; x \in \Omega : P(X = x) > 0$.
2. $\forall\; t \in S\;\; x \in \Omega$: $P(x_i/x_j, j \in S - \{i\}) = P(x_i/x_j, j \in Ni)$.

More details about MRF are stated in [19].

In this paper the fitness function used for pixels is the a-posterior probability $P(x/y)$ that follows a Gibbs distribution defined by:

$$P(x/y) = e^{-U(x/y)} \big/ Z_y \qquad (1)$$

Where Z_y is the normalization constant and $U(x/y)$ is the energy function given in Equation (2):

$$U(x/y) = \sum_{t=1}^{MN}\left[\ln\left(\sqrt{2\pi}\,\sigma_{xt}\right) + \frac{\left(y_t - \mu_{xt}\right)^2}{2\sigma_{xt}^{\;2}} + \sum_{r \in N_t}\left(\beta\delta\left(x_t, x_r\right)\right)\right] \qquad (2)$$

Where β is a positive model parameter that controls the homogeneity of the image segments and σ_{xt}^2 and μ_{xt} are respectively variance and mean of a segment. Also $\delta(a, b) = -1$ if $a = b$, 1 if $a \neq b$.

We can define image segmentation as the estimation of configuration label x which maximizes the fitness function $P(x/y)$ called the MAP estimation.

4. The Proposed Model

Our model starts with initial oversegmented images. The sandpile model is used for merging similar segments and EO is applied to adjust the labels of pixels.

The detailed information about the method is as following:

4.1 Concepts

At First, colour reduction operation is done to reduce the number of colours in a given image in order to limit the configuration space.

Then the searching strategy is started to employ the energy function in markov random field model for pixels given in Equation (2). The aim of the algorithm is to minimize this energy function.

The first part of the energy function is related to a pixel in the segment it belongs. And the second part of the energy function is related to the neighbours of each pixel.

In this work we use second order neighbourhood system in MRF model in which eight nearest neighbours of each pixel is used.

The more the difference of the pixel’s colour with the mean colour of the segment it belongs, the more the energy function and the less the fitness of that pixel, which is stated in the first part of the energy function. So the algorithm tries to assign a label to a pixel that increases its fitness.

When we say a pixel belong to a segment, it means the label of the pixel is the same as the label of some other pixels in that segment.

As we mentioned earlier, the second part is related to the labels of the neighbours of a pixel. If two nearest neighbours have same labels (shows that they belong to same segment) then δ = -1 and if two nearest neighbours have different labels (such as pixels in the boundary of a segment or pixels that seems like noise because they have few adjacent pixels with the same label) then δ = +1.

Parameter δ is calculated for the eight nearest neighbours of a pixel. The more the δ for a pixel the less the fitness of that pixel and it is more likely to be mutated by the algorithm.

4.2 Increasing avalanche power

For increasing the speed of the algorithm we were to enhance the self organizing power by running strong avalanches in the system. When a pixel is selected to be mutated by changing its label, all the neighbours of this pixel and also the neighbours in the further level and etc (we also used neighbours in a few further level) who have the same colour as this pixel (after colour reduction step) will change their labels to the new label as this pixel. Consequently, avalanches in different shapes, circle, line etc (depend on the number of the neighbours who changed their labels) occur in the system.

4.3 Simulate the sand pile model in the system

Inspiring by sand pile model as mentioned earlier, while selecting some pixels for mutation and change its label and some neighbours (to a few further level of neighbourhood) , the number of pixels which belong to a specific segment ‘a’ will decrease and the number of pixel for segment ‘b’(the segment that these pixels change their labels to) will increase. For each segment that lost some pixels we will increase its grain as much as the number of pixels it has lost. Also, for each segment who gains some pixels we will decrease its grain to the number of pixels it gains. If the grain of a segment reaches to a given threshold, it is ready to extinction. The grain of this segment is divided among its neighbours and it should be merged with one of its neighbours (the neighbour who has more similarity in colour).

This process is repeated for each segment that its grain reaches to the threshold.

The purpose of adding/subtracting the grain of a segment is to make a difference between a good and a bad segment. If a segment is a bad segment, so in some different steps of the algorithm it will lose some pixels (in each iteration some pixels will change their labels to optimize the energy function.)

After merging similar segments, EO tries to adjust pixels by finding better labels for some pixels in order to optimize the energy function defined for pixels in equation (2). So, in this step, the algorithm just works on pixel level.

We consider site labels as species and the image as lattice- system as in bak–sneppen model.

The fitness value of species x_i is λ_i given by equation (3) that uses the energy function in equation (2):

$$\lambda_i = P(x_i/x_j, j \in N_i) \qquad (3)$$

The pixel level EO algorithm (after merging similar segments by sandpile model) is presented in Fig. 2.

1. *An initial solution $x(0)$ (segmented image after merging step)*

For $t = \{1, ..,MN\}$ Do compute λ_t. Compute $F = U(x/y)$.

2. *Let $x_{best} = x$, $F_{best} = F$ and Iteration = 1.*
3. *Rank the pixels according to their fitness (the worst site label has rank 1 and the best site label has rank MN)*
4. *For s = 1..MN Do*
   a. *Compute probability $P_s \propto s^{-\tau}$ where τ is a parameter*
   b. *Generate a uniform random number μ_s in [0, 1].*
   c. *If $\mu_s <= P_s$ Then modify the label of the site s to one of it's nearest neighbors label.*
5. *For t = 1...MN Do evaluate λ_t of x.*

Compute $F = U(x/y)$.

6. *If $F < F_{best}$ Then $x_{best} = x$ and $F_{best} = F$.*
7. *Iteration = Iteration +1.*
8. *If (Iteration $\leq$ a given number of iterations) Then goto 3.*
9. *Output x_{best} and F_{best}.*

Fig. 2: Pixel level EO algorithm

5. Results

Results of the algorithm on different evolutionary steps are displayed in the following figures. The problem parameters are tuned as: threshold =40 for grains and β = 0.5 and τ = 1.9 for EO.

For representing the propagation of grains in different segments, in Fig. 3.b a simulated oversegmented image with large extra segments is shown. In different steps of the algorithm some pixels are mutated by changing their labels. So some pixels are propagating to different segments.

Merging similar segments occurs when a segment grain reaches to the threshold value (Fig. 3.c, 3.d). Also EO algorithm will adjust improper labels of pixels by minimizing the energy function.

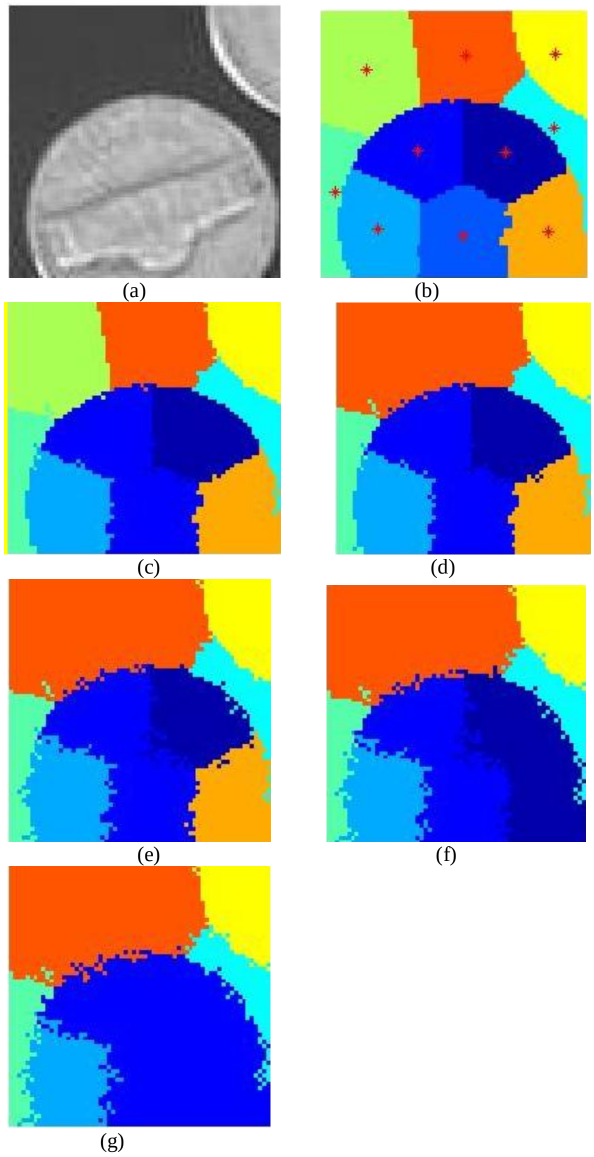

Fig. 3: Evolutionary steps of the proposed method a)original image b) simulated oversegmented image c, d, e, f, g)propagation of grains into different segments and merging similar segments.

In Fig. 4 real oversegmented image is given to the algorithm and the quality of segmentation is improving gradullly by merging similar segments while identifying the main segments in initial image and then adjusting pixels' labels.

Although the quality of segmentation is not as good as some other segmentation method, but segmentation can be a good framework for showing the concept of self organization.

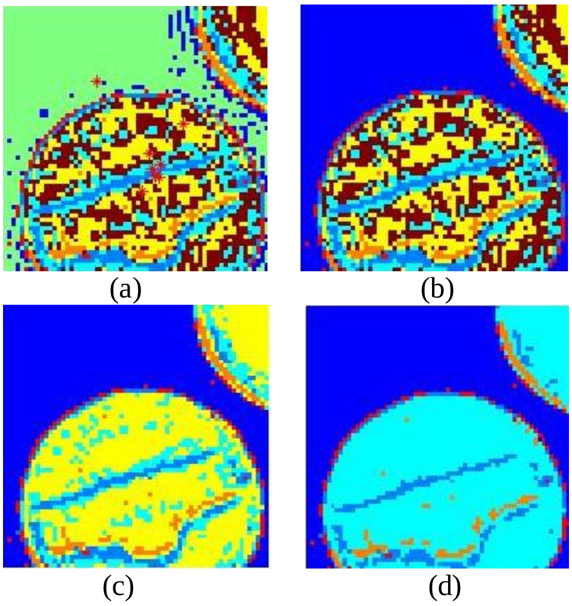

Fig. 4: Evolutionary steps of the proposed method for real oversegmented image a)oversegmented image b, c, d)propagation of grains into different segments and merging similar segments. (the labels of pixels will be adjusted by EO.

6. Conclusion

In this paper, we present a new application of image segmentation for sandpile model. At first, over-segmented images are given to the algorithm. Inspired by sandpile model, similar segments are then merged and the labels of pixels are adjusted by EO. At present, this paper just gives the basic idea of applying sandpile model in image segmentation. Although results are not necessarily as strong as other segmentation method, this paper shows that self organized critically concept can indeed be applied to the field of imaging. Such concept poses the possibility of fast convergence and lowered computational complexity in addition to high segmentation performance. We hope to address this issue in our future work.

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