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Composing local and global behaviors: Higher performance of spin glass based portfolio selection

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ABSTRACT

The basic challenge in optimization is how to navigate through the many non-optimal and mediocre solutions toward the few globally optimal solutions, amidst the growing problem size and computation complexity. If the proximity to an optimal solution could be measured, a desirable technique could be one that navigates speedily, even if crudely, when an optimal solution is not likely to be next; and accurately, even if slowly, otherwise. In this paper, we propose a technique based on spin glass paradigm that uses the above heuristic to solve the classic portfolio selection problem. Study of spin glass paradigm reveals that limiting each spin's interactions to its local neighborhood increases the computational speed of the algorithm, but also introduces an error in performance measure. In contrast, extending each spin's reach globally provides an accurate measure of performance, but slows down the glass computations. Theoretical analysis reveals a decision threshold by which speedy versus accurate navigation, i.e. local versus global glass behavior, can be alternated. The resulting algorithm is then applied to five different world stock market portfolio selection problems consisting of Hang Seng, DAX 100, FTSE 100, S&P 100, and Nikkei. These results demonstrate utility of the hybrid local–global behavior and appropriateness of the proposed decision threshold. Specifically, the results of experiments show faster convergence without a significant loss of accuracy in reaching globally optimal solutions.

1. Introduction

Ising model is a network of spins in which spins interact magnetically, and consistently change their values to achieve a lower level of energy. When the system is in minimum energy (or temperature) state, there is no significant variation in spin values and the system is said to have reached ground state (the state with minimum energy). This model has many characteristics among which are non-exponential growth of ground states with an increase in the number of spin bonds, effectiveness of environmental factors such as temperature on network behavior and ability to achieve the optimum state at variant temperatures. Many optimization problems can be solved according to these characteristics as distributed.

Since the overall network of spins represents only one solution, there is a great promise in computational efficiency when compared with other population based such as Genetic Algorithm (GA) and Particle Swarm Optimization (PSO), agent based such as Differential Evolution (DE) and stochastic approaches such as Simulated Annealing (SA). It is different from other distributed frameworks such as Hopfield Neural Network (HNN) since spins' paradigm of interaction does not have to be fully connected, i.e. the neighborhoods of interactions can expand or collapse. In addition to all the above characteristics, spin glass structure is self-organizing with avalanche like property. Its behavior is non-ergodic in freezing temperature, where below that temperature the system cannot escape from the minima of the hierarchically disordered energy landscape. Also spin glass has critical temperature that is named phase transition temperature where the spin glass exhibits more typical magnetic behavior before and after this temperature.

The convergence speed of this model, like many other heuristic algorithms is low; so, with the replacement of local behavior, speed can be increased. According to a hypothesis expressed by Bauke and Mertens [1], local behavior of spins leads to random changes in glass' energy. Experiments by Bovier and Kurkova [2] show the correctness of Mertens hypothesis. This concept indicates that using only local behavior may not be sufficient for solving engineering problems. However in 2010, Vafaei Jahan and Akbarzadeh-T. [3] showed that the speed of convergence can be improved by local glass behavior and coordinated random alternations of its

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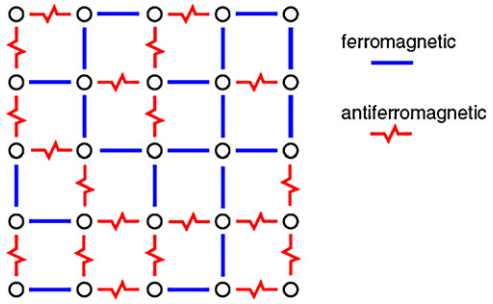

Fig. 1. A two-dimensional spin glass with bond disorder. Spins are placed on the sites of a regular grid. They interact with their neighbors, the interaction is random, either ferromagnetic (straight lines) or anti-ferromagnetic (jagged lines) [10].

structure. It was shown that the accuracy of their approach depends directly on the low level of bonding energy between spins, which can be restrictive for solving engineering problem.

Unlike the local behavior of spin glasses in [3], this paper presents a new algorithm that uses local behavior for increasing rate of convergence and uses global behavior for increasing the accuracy to achieve global optimums. This algorithm detects the appropriate threshold for spin glass behavior changes from local to global and hence controls the convergence speed versus accuracy. In other words, this algorithm reveals a decision threshold by which local (short range) versus global (long range) glass behavior can be automatically alternated.

Following Section 1, spin glass model is discussed in Section 2. All types of spin glass behavior are discussed in Section 3. The portfolio selection problem is described in Section 4. In Sections 5–7, portfolio selection problem is solved using an algorithm based on spin glasses and an analysis of its convergence speed and accuracy is provided. Sections 8 and 9 illustrate the performance of this method on benchmark data with analyzing reliability test and Pareto frontier. Finally, conclusions appear in Section 10.

2. Spin glass model

Some aspects of the approach are similar to authors' earlier work in [3] and are repeated here for completeness. A suitable theoretical model describing spin glasses consists of N spins placed on the regular sites of a d-dimensional lattice with linear extension L, e.g., quadratic ($N=L^2$) or cubic ($N=L^3$). The spins interact ferromagnetically or antiferromagnetically with their neighbors. A small example is shown in Fig. 1.

The energy of such a network comes from two contributions [7,12] and can be written as below:

$$E(\{x_i\}) = \left[-\frac{1}{2}\sum_{i=1}^{N}\sum_{\langle i,j\rangle}^{m} x_i J_{ij} x_j\right] + \left[-\sum_{i=1}^{N} h_i x_i\right] \quad (1)$$

where $E(\{x_i\})$ is the energy of all spins. The sum i, j runs over all pairs of nearest neighbors and m is the number of nearest neighbors of each spin i interacts with spin j that can be $m=4$ in Von Neumann cellular automata (CA) or $m=8$ in Moore CA in short range connection or $m=N$ in long range connection. J_{ij} denotes the strength of the bond connecting spins i and it's nearest neighbor j. $J_{ij}>0$ describes a ferromagnetic interaction, while $J_{ij}<0$ describes an anti-ferromagnetic interaction. The quantity h_i is the external field acting on spin i and describes the energy due to the spin's orientation. Also, the factor 1/2 corrects for double counting of the interaction between every two neighboring spins. Here the task is to find a spin configuration x_i that minimizes the energy of the spin glass, given $\{J_{ij}\}$, $\{h_i\}$.

3. Spin glass behavior

Each spin can alter its own value. Effect of this change on its neighbors defines different behaviors of the glass. In this article, two general types of behavior for each glass are considered:

1. **Global behavior (G)**: In this behavior, due to spin–spin interactions, variation in the value of one of the spins affects all of the spins and consequently the total energy. In this case, an increase or decrease in the spin energy yields a similar change in the overall energy of the network. While this behavior is desirable, it imposes exponential time complexity since the number of interactions (bonds) grows with the size of the glass.
2. **Local behavior (L)**: In this behavior, if there is a variation in the value of a spin due to the forces between the spin and its neighboring spins, this variation is only propagated to the spin's immediate neighbors. In this case, the energy variation of the spin and its neighbors would not necessarily correspond to the global energy changes. While global conclusions cannot be made with this type of local behavior, its computational complexity remains linear.

Considering the above two local and global behaviors as well as global versus local definition of neighborhoods, four types of behaviors can be expected [3]:

- **Global behavior with $m=N$**: In this condition, cost function equals the overall spin glass energy function. In [3], it is shown that spin glass tries to find states with lower energy (toward ground states). Therefore, optimum portfolio selection equals optimum spin glass states. The advantage of this behavior is its accuracy in obtaining the optimum portfolio, but it is slow for large N.
- **Local behavior with $m=N$**: This behavior is concerned with the local energy of each spin when $m=N$. Since all of the spins in the glass are neighbors of a given spin, local behavior is similar to global behavior. However, due to the decrease in local energy, the rate of convergence can be slower than global behavior.
- **Global behavior with $m<N$**: Under such condition, the total energy variation of the network is calculated based on the variation in each spin, but the variation in each spin causes a variation only in its neighbors and not all of the spins. Therefore, the optimum cost function does not necessarily correspond to the optimum spin glass state. The migration and elitism operators modify the placement of spins in a way to enable exploiting the glass structure to find the optimum portfolio. These two migration/elitism operators are discussed in [3] and also in Section 7.1.
- **Local behavior with $m<N$**: In this condition, neighbors of an altered spin are adjusted, but this variation is not further propagated to all spins in the glass. This behavior is the most amenable to parallel processing and is quite fast; but it also has two difficulties. Firstly, $m<N$ causing a deviation in the value of cost function and spin glass energy (energy level difference); and accordingly, a decrease in one may not be accompanied by a decrease in the other. Secondly, the local activity of spins does not necessarily lead to global optimization.

4. Portfolio selection problem

Let us consider the Markowitz mean-variance model [4] for the portfolio selection problem (this problem plus its constraints is one

of the nondeterministic polynomial (NP) complete problems) as stated below,

$$\text{Min} \quad \sum_{i=1}^{N}\sum_{j=1}^{N} x_i \sigma_{ij} x_j \tag{2}$$

$$\text{Max} \quad \sum_{i=1}^{N} \mu_i x_i \tag{3}$$

$$\text{Subject to} \quad \sum_{i=1}^{N} x_i = 1 \tag{4}$$

$$0 \le x_i \le 1, \quad i = 1, \ldots, N \tag{5}$$

where N is the number of assets, μ_i is the mean return of asset i, and σ_{ij} is the covariance between returns of assets i and j. The decision variable x_i represents the fraction of capital to be invested in asset i. Eqs. (2) and (3) are two cost functions that should be solved with constraints (4) and (5). μ_i is the mean return of asset i in n intervals of time, i.e. $\mu_i = \sum_{t=1}^{n}(W_{ei}(t) - W_{bi}(t))/W_{bi}(t)$, where W_{bi} is the ith asset value at the beginning and W_{ei} is the ith asset value at the end of each interval.

Solving this problem with multiobjective optimization methods has been presented in [3]. A feasible solution of the portfolio selection problem is an optimal solution if there is no other feasible solution improving one objective without deteriorating the other. Usually, multiobjective optimization problems such as those in [5] have multiple non-dominated optimal solutions. This set of solutions form what is called an efficient frontier. For the problem defined in Eqs. (2)–(5), the efficient frontier is an increasing curve that gives the best tradeoff between mean return and variance (risk).

In this paper we change the multi-objective problem into a multimodal problem with single objective function as follows,

$$\text{Minimize} \quad \lambda \cdot \left[\sum_{i=1}^{N}\sum_{j=1}^{N} x_i \sigma_{ij} x_j\right] + (1-\lambda)\cdot\left[-\sum_{i=1}^{N} \mu_i x_i\right] \tag{6}$$

$$\text{Subject to} \quad \sum_{i=1}^{N} x_i = 1 \tag{7}$$

$$0 \le x_i \le 1, \quad i = 1, \ldots, N. \tag{8}$$

In Eq. (6), let $\lambda \in [0, 1]$ be the risk aversion parameter. If $\lambda = 0$ then Eq. (6) represents maximum portfolio mean return (without considering the variance) and the optimal solution is formed only by the asset with the greatest mean return. The case with $\lambda = 1$ represents minimizing the total variance associated with the portfolio (regardless of the mean returns) and the optimal solution typically consists of several assets. Any value of λ inside the interval $[0, 1]$ represents a tradeoff between mean return and variance, generating a solution between the two extremes, $\lambda = 0$ and 1.

5. Solving portfolio selection problem using spin glass paradigm

To present our method, we initially map the portfolio selection problem into a spin glass computational model. The glass' ground state is found by considering the objective function of the portfolio selection problem, Eq. (6), and comparing it with the spins energy function of the spin glass model, Eq. (1). We obtain the values for the interaction strength as follows:

$$J_{ij} = -2\lambda\sigma_{ij} \tag{9}$$

$$h_i = (1-\lambda)\mu_i \tag{10}$$

The decision variable x_i represents the proportion of capital to be invested in asset i; accordingly in spin glasses, x_i is the state of spin i. So the problem of portfolio selection can be solved by minimizing the mapped function as in Eq. (1).

6. Spin glass energy with different neighborhood types

As observed in different experiments, the local behavior initially decreases the cost function and then remains constant or increase a little. This means, while glass passes from the initial or inappropriate state, local behavior can be a desirable behavior. However, the glass behavior after that is undesirable, causing the glass to move away from its optimal solution. Hence, it would be more desirable to switch to global behavior. Theorem 1 determines a decision threshold by which the switch between local and global behaviors can occur.

Theorem 1. *Consider the ground state energy $E(m)$ of a spin glass with N spins, where $m \le N$ is the number of nearest neighbor spins. Then, the following upper bound for energy difference can be established as follows:*

$$E(m) - E(m') \le -\frac{J}{2N}(m - m') \tag{11}$$

where J is weakest glass bound, s_i and s_i' are the amounts of ith spin in glasses with m and m' nearest neighbors.

As mentioned in previous section, optimum portfolio equals optimum spin glass state when $m = N$. Therefore, the value of cost function at each moment equals the value of spin glass energy function. Also according to the mean field theory [6], the magnetization of the spin glass equals:

$$g = \frac{1}{N}\sum_{i=1}^{N} s_i \qquad \Delta s_i = s_i - g \tag{12}$$

where g is the overall magnetization, i.e. the average value of spins of the network; and $\Delta s_i = s_i - g$ is the difference of each spin from the overall magnetization (average). Thus with regard to spin glass energy function in Eq. (1):

$$E(\{s_i\}) = -\left[\frac{1}{2}\sum_{i,j}^{m} J_{ij}.(g + \Delta s_i)\cdot(g + \Delta s_j) + \sum_i h_i \cdot s_i\right]$$

$$\approx -\left[\frac{1}{2}\sum_{i,j} J_{ij} g^2 + \frac{1}{2} g \sum_{i,j} J_{ij}(\Delta s_i + \Delta s_j) + \sum_i h_i s_i\right] \tag{13}$$

where the second order interactions $-(1/2)J_{ij}\,\Delta s_i\,\Delta s_j$ are assumed to be negligible. Since $\Delta s_i = \varepsilon$ and the change in neighboring spins is correspondingly $\Delta s_j = \varepsilon/m$, $-(1/2)J_{ij}\,\Delta s_i\,\Delta s_j = -(1/2m)J_{ij}\varepsilon^2$. Since ε is assumed to be small, and J_{ij} is also bounded, the approximation in Eq. (14) is reasonable [6]. Furthermore, in portfolio selection problems such as those considered here, J_{ij} is also small.

If J_{ij} is replaced by its upper bound (weakest link) $J \cong Max(J_{ij})$ and h_i by its highest amount (maximum external field) $H \cong Max(h_i)$, an upper bound for the spin glass energy can be established by Eq. (14), as below:

$$E(\{s_i\}) \le -\left[\frac{1}{2}\sum_{i,j} Jg^2 + \frac{1}{2} g \sum_{i,j} J(\Delta s_i + \Delta s_j) + \sum_i H s_i\right]$$

$$\le -Jg^2 N_B - Jgm\sum_i \Delta s_i - H\sum_i s_i \le -Jg^2 N_B - Jgm\sum_i \Delta s_i - H \tag{14}$$

where N_B is the total number of bonds in the network and is upper bounded by $N_B \leq (mN/2)$. $N_B \leq (mN/2)$ when the spin network is round, i.e. boundary spins in the extreme edges form a bond. Substituting $\Delta s_i = s_i - g$ in Eq. (14), we have:

$$E(\{s_i\}) \leq -Jg^2 N_B - Jgm\sum_i \Delta s_i - H \leq Jg^2 N_B - Jgm\sum_i s_i - H \quad (15)$$

In the proposed algorithm, due to constrained nature of the Markowitz problem in Eq. (3), the magnetization of the network is always constant and equals:

$$g = \frac{1}{N}\sum_{i=1}^{N} s_i = \frac{1}{N} \quad (16)$$

Therefore the energy difference for the network with $m > m'$ equals:

$$\Delta E = E(m) - E(m') = \frac{J}{N^2}(N_{B'} - N_{B''}) - \frac{J}{N}(m - m') \leq -\frac{J}{2N}(m - m') \quad (17)$$

Hence, the difference between two levels of energy of a glass with different numbers of spin neighbors for portfolio selection problem is independent of mean return of assets h_i and directly depends on covariance between returns J_{ij}. Also, as spin neighborhoods grow, this difference also decreases. For example, for $m = m' = N$, this difference is trivially equal to zero. In local search, however, shrinking neighborhoods is favored. Hence, either J should be small or the number of spins should large ($N \gg 1$), if the local behavior is to approximately lead to finding the optimum solution.

The above analysis applies to maximization functions with only a small change in the proof, i.e. $J \cong Min(J_{ij})$ and $H \cong Min(h_i)$.

6.1. Proposed spin glass algorithm

In following Theorem 1, for minimizing Eq. (6) with regard to constraints (7) and (8), we first randomly place the possible assets into a $L_1 \times L_2$ lattice-like structure such that $N = L_1 \times L_2$, where N is the number of assets. All of the spins in this structure are initialized to $1/N$. Therefore, we can select the best assets using the following algorithm:

Algorithm 1 *(Composed spin glass optimization).*

Begin

1. Initialize m and α, begin with local behavior.
2. Select spin i randomly
3. Change the state of spin i by ε (very small change) and adjust all the nearest neighboring spins for satisfying constraints (7) and (8).
4. Calculate the energy of the selected spin and its neighboring spins $\left(E_{new} = \sum_{i=1}^{m} E_i\right)$
5. $\Delta E = E_{new} - E_{old}$
6. If $\Delta E < 0$ then accept this change else
7. If $\Delta E > 0$ then accept this change with probability $e^{-\Delta E/T}$
8. If rejected, exchange spin location with one of its neighbors by either migration or elitism. (For migration, exchange the spin with another randomly chosen spin in the glass. For elitism, if the spin has higher level than any of the left or upper-left corners, they exchange places.)
9. If $E(N) - E_{new} > -\frac{\alpha J}{2N}(N - m)$, then change to global behavior.
10. Continue this process with decreasing temperature till either ΔE remains near 0 for several iterations (i.e. the system has reached steady state), or T has reached near 0 (system has cooled).

End

In the above algorithm, E_{old} and E_{new} represent the total energy of the network before and after applying changes, $E(N)$ is the value of cost/energy function based on current spin glass state with N spins. T is the temperature of the glass. T is initially very high in order to enable diverse search. $0 \leq \alpha \leq 1$ is the coefficient of variation. If $\alpha = 0$, the behavior of the glass is global, and the higher α corresponds to more local behaviors. As will be shown in the section on experiments, the best value of α is usually between 0.3 and 0.5. In this paper, α is equal to 0.3. ε is a small value, here 0.05, that shows the change of spin state in each spin flip. Our analysis indicates that ε is inversely proportional to $\bar{J}$ (mean value of J_{ij}), particularly when $\bar{J}$ is large. For smaller $\bar{J}$, the value of ε becomes less important since the spin movement has less global effect. Overall, our experiments indicate that a value of 0.05 is a good value across all benchmarks.

According to Algorithm 1, a spin is chosen randomly at every flip and the value of the selected spin is increased by ε. Accordingly, the value of neighboring spins changes to meet the constraints (7) and (8). The amount of energy is then estimated. If there is a decrease, the change is accepted; otherwise it is only accepted with probability $e^{-\Delta E/kT}$ with $k = 1$ [7].

At each iteration, when the difference between cost function value and the energy of glass (m-neighbor glass) is more than $(-\alpha J/2N)(N - m)$, the behavior of glass should be changed from local to global. This procedure continues until either the minimum energy is achieved or system is completely cooled.

For the heating and cooling schedule, procedures related to SA are used, as in [8,13–15]. To do so, the temperature of the network is considered to be initially set to $T_0 = 1$ (at high temperatures all states can occur). Each time the changes are applied, the temperature is gradually decreased until it reaches near zero. Temperature variations can be governed by the following formula,

$$T(n) = \frac{T_0}{n^2}, \quad n \geq 1 \quad (18)$$

where n represents the number of epochs. The stop condition of algorithm is the repetition of a solution for a certain number of steps. For example all experiments here have concluded when reaching same results for ten steps within a tolerance of 10^{-7}.

6.2. Constraint satisfaction

In portfolio selection problem, two constraints (6) and (7) have brought about some considerations. In order to satisfy the constraint (6), i.e. to keep the total amount of spins constant at 1, when any spin is increased by ε ($x_i := x_i + \varepsilon$), ε/m is subtracted from the spin's neighbors ($x_j := x_j - (\varepsilon/m)$ where $j = 1, \ldots, m$). If $x_i \geq 1$ then $x_i := 1$ and the extra value is subtracted from ε; alternatively, if for each neighbor $x_j - (\varepsilon/m) \leq 0$, then $x_j := 0$ and the value difference is added to x_i. According to the above explanation constraint (7) is also satisfied.

7. Experimental results

In order to verify the effectiveness of the above algorithm, the benchmarked "standard efficient frontier" (Pareto Front) is compared with the efficient frontier resulting from the proposed method.

Experiments on the benchmark data were originally performed in [9]. These benchmark data are presented in text file format as follows:

Number of assets (N); and for each asset i ($i = 1, \ldots, N$): *mean return* as well as *standard deviation of return*; for all possible pairs of assets: i, j, *correlation* between asset i and asset j. The above data were taken from five major stock exchange markets, during the time period extending from March 1992 to September 1997. These

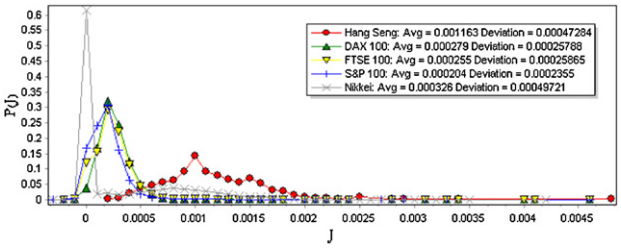

Fig. 2. Probability density functions for covariances between assets in 5 major stock markets from 1992 to 1997 from data in [9].

five stock exchange markets include Hang Seng in Hong Kong (31 assets), Deutscher Aktien Index (DAX 100) in Germany (85 assets), Financial Times London Stock Exchange (FTSE 100) in Britain (89 assets), Standard & Poor's (S&P 100) in USA (98 assets), and Nikkei in Japan (225 assets). Probability density function (pdf) of covariance $P(J)$ of each stock market for the given data has been shown in Fig. 2.

As can be observed, the $P(J)$ of the five given stock markets have small mean and variance. Standard efficient frontier for each of these five stock markets in the available time period is characterized by mean return as in Eq. (3) and variance of return as in Eq. (2). Fig. 3 illustrates these efficient frontiers.

Various tests concerning the analysis of spin glass behavior are considered here. First, spin glass convergence is analyzed with $m = N$ and $m < N$ based on local or global behaviors of the spin glass and assessing the possibility of achieving the optimum solution. Second, the resulting efficient frontier is compared with the standard efficient frontier [9]. Third, reliability test analysis of the algorithm is performed.

All of the experiments are performed using Borland Delphi 6.0 running on a Pentium 2.4 GHz PC, under Windows XP operating system. It should be mentioned that each spin flip equals performing the algorithm once and each epoch equals 100 flips.

7.1. Migration and elitism operators

As mentioned in paper [3], it is shown that due to low covariance in portfolio data, we can use spin glass with local behavior to obtain the optimum solution without a significant loss of accuracy. Therefore, it suggests using genetic algorithm operators such

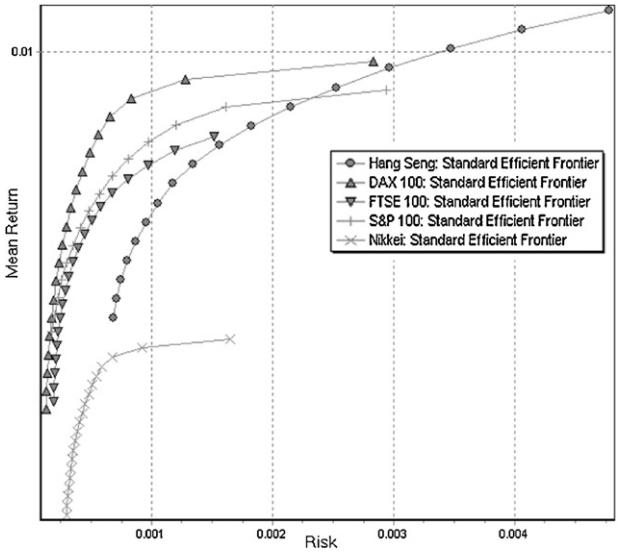

Fig. 3. Standard efficient frontier for benchmark data from 5 major stock markets [9].

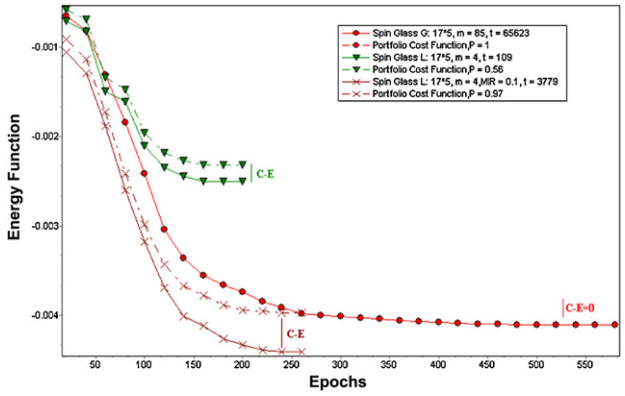

Fig. 4. Comparing local, global and composed behaviors of spin glass for glasses with 32 spins. P shows the probability of finding ground state.

as migration and elitism in order to increase the rate of convergence. In paper [3], migration refers to the transferring of selected spin to another random location which is done with the rate *MIR*. Elitism gradually and locally moves spins with higher spin level s_i toward each other and toward the upper left corner of the glass. This is a local operation by which spins compare their own energy with that of their upper and upper-left neighbors. If it has higher spin level than any of the above two neighbors, they exchange places. This is done with an elitism rate of *ELR* for the selected spins. The replacement of spins by the elitism operator causes the elite spins to be placed next to each other and this causes a competition among the elite spins and hence increases the rate of convergence significantly.

7.2. Composing local and global behaviors

As mentioned in Section 3, the local behavior speeds up the convergence and global behavior increases the accuracy. As Theorem 1 shows, there is an upper bound on the difference between cost function and glass energy. Hence, by composing these two behaviors and selecting an appropriate α, speed and accuracy can be both obtainable. In other words, first, local behavior passes initial and inappropriate state rapidly, then, global behavior searches optimal states in much less states carefully. This strategy maintains both accuracy and speed to find ground state.

In Fig. 4, two curves for each test are shown. One is for glass' energy and one is for cost function value. When the behavior is global both these two curves are same. But when the behavior is local, the difference between them causes two different curves. As can be seen in all tests, the portfolio cost function's curve is higher than spin glass' energy curve. This is because when the behavior is local, i.e. $m < N$, $N - m$ non-neighbor spins are not accounted for in the energy function. So, this causes the difference between cost function and glass' energy and reduces the probability of finding ground states.

As observed in Fig. 4, the two curves (denoted by circles) present cost function (dotted) and actual glass energy (solid) based on global behaviors and have the best performance in terms of accuracy ($P = 1$ shows the ground state that has been found with global behavior accurately). In this behavior, the difference between cost function and glass' energy is zero and both curves are same. In contrast, for the second curve (denoted by triangles), the behavior is local; hence the glass is placed in its local optimum and a significant difference between cost function and glass' energy is formed ($P = 0.56$ shows the probability of finding ground state is 0.56).

In the third curve (denoted by stars), the behavior is still local and hence there is still a difference between cost function and glass' energy. This curve uses migration operator with rate 0.1 and the

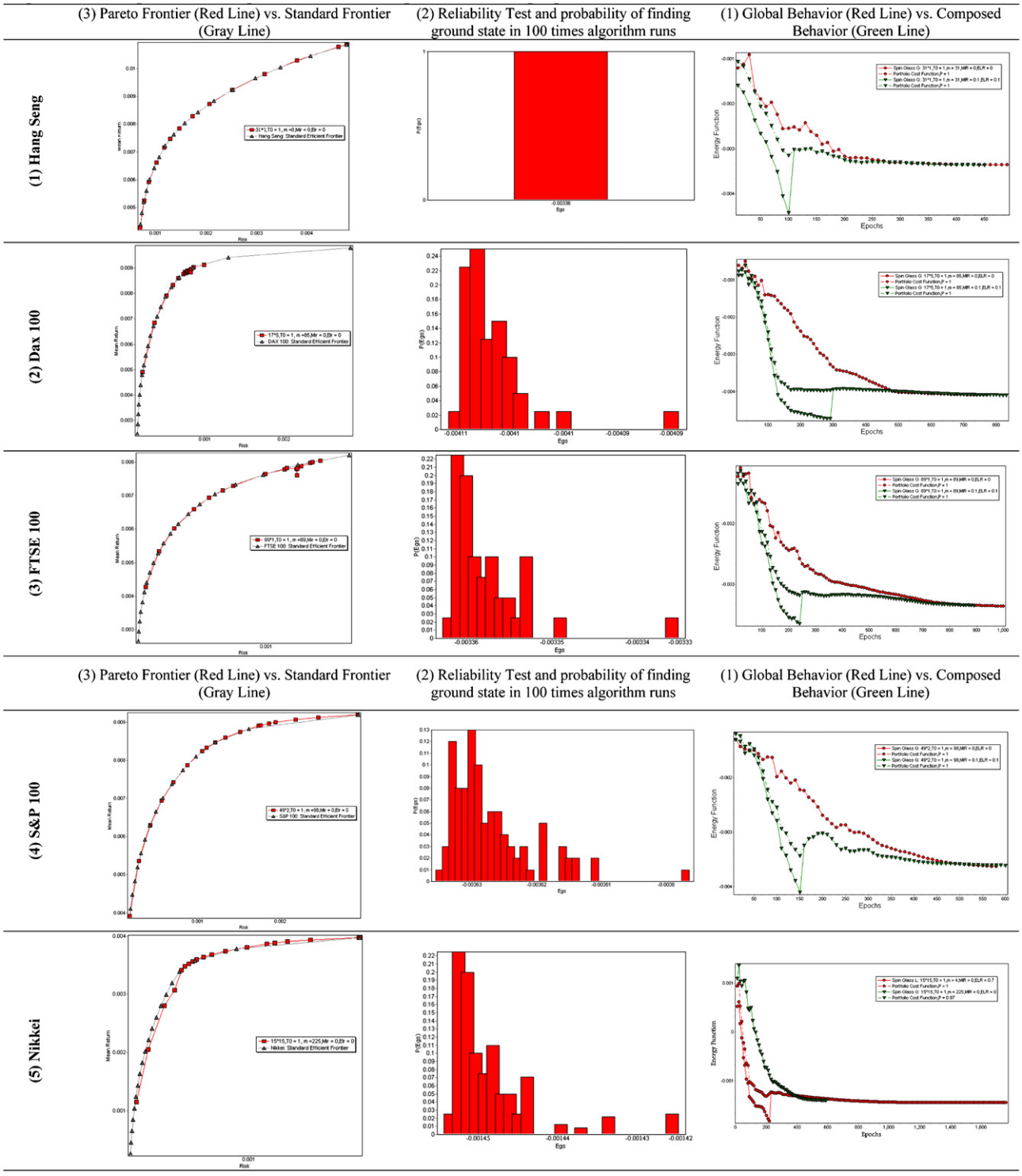

Fig. 5. The comparison between global behavior and composed behavior of spin glass for five stock market [9].

Table 1
Comparison between global ($m = N$), local ($m = 8$) and composed behaviors (the ground state of glass for each stock is calculated 100 times and the average and variance are listed in column "Ground state energy" and "Mean absolute error").

Stock	Global behavior [3]			Local behavior [3]			Composed behavior		
	Run-time (ms)	Ground state energy	Mean absolute error	Run-time (ms)	Ground state energy	Mean absolute error	Run-time (ms)	Ground state energy	Mean absolute error
Hang Seng ($N = 31$)	4626	−0.00336	7.2×10^{-12}	2180	−0.0029	4.6×10^{-4}	3620	−0.00336	8.4×10^{-12}
DAX 100 ($N = 85$)	65268	−0.00412	2.6×10^{-11}	17659	−0.0038	3.2×10^{-4}	16005	−0.00412	1.1×10^{-11}
FTSE ($N = 89$)	75631	−0.00335	2.0×10^{-11}	31981	−0.0029	4.5×10^{-4}	21135	−0.00335	2.1×10^{-11}
S&P ($N = 98$)	81231	−0.00363	1.76×10^{-11}	32039	−0.0031	5.3×10^{-4}	24402	−0.00363	1.71×10^{-11}
Nikkei ($N = 225$)	418140	−0.00145	2.76×10^{-10}	81321	−0.0012	2.5×10^{-4}	212120	−0.00145	1.67×10^{-10}

Table 2
Comparison between SG, GA, TS, SA and NN. The ground state of glass for each stock is calculated 100 times and the average of convergence time (run-time) and accuracy are listed. The experiment result numbers (only accuracy numbers) mentioned for GA, TS, SA, NN are extracted from paper [16].

Stock		SG	GA	TS	SA	NN
Hang Seng ($N=31$)	Accuracy	0.25	1.1321	1.1237	1.1203	1.2316
	Run-time (s)	3	51	19	18	390
DAX 100 ($N=85$)	Accuracy	0.2618	2.4457	2.6668	2.3896	1.5776
	Run-time (s)	16	162	45	62	1069
FTSE ($N=89$)	Accuracy	0.6303	0.7310	0.7357	0.9512	1.2513
	Run-time (s)	43	148	57	83	1106
S&P ($N=98$)	Accuracy	0.475	1.3236	1.3130	1.7251	1.7922
	Run-time (s)	31	178	50	49	1211
Nikkei ($N=225$)	Accuracy	0.9027	1.1415	0.5510	0.5458	1.4737
	Run-time (s)	218	570	590	391	2793

behavior is better ($P=0.97$). As observed, the probability of finding ground state is near 1 and the time (t) to reach ground state is $t=3779$ ms, see Table 1. This time is shorter than global behavior, since the behavior is local the glass placed in local optimum near the ground state. In this situation we can change the behavior from local to global. Algorithm 1 helps us to escape from local optimum and achieve global optimum.

Fig. 5 shows the experimental result of Algorithm 1. In Column (1), spin glass' behavior is initially local and after several epochs the behavior is changed to global. The comparison of run time between global, local and composing behaviors is shown in Table 1. As observed, the value of ground state is the same for both globally and composed methods but the run-time of composing method is better.

7.3. Comparison with other heuristics

Now let us take a look at some numerical results in Table 2. This table is complement of Table 1 in paper [16] that shows the comparison of Neural Network (NN) approach with GA, Tabu Search (TS) and SA. Table 2 allows some kind of comparison between our spin glass (SG) method results and those of paper [16]. In Table 2, accuracy of the mentioned methods and their run-time are shown. (The run time of the proposed method and the current methods in paper [16], due to the differences in computers and similar simulation software, is not the same. Because, the authors implement the GA, TS, SA and NN, according to the author's understanding of the paper [16]; in Table 2 the new run time results have been listed. Some differences in the experimental run time results are observed but the conclusion seems generally correct) Accuracy shows the degree of closeness of measurements of an objective function to its actual value that is shown as "mean absolute error" to actual objective function (actual cost function) value. With regard to the computation times, SG is the most efficient algorithm followed by SA, TS, GA and finally NN. When comparing SG with other heuristic methods, SG improves considerably (except for the last stock market Nikkei) accuracy and run time for all different stock markets. This significant increase in convergence speed is except for the use of parallel facilities of spin glass that is completely discussed in paper [3].

8. Reliability test

Test of reliability is performed by running the algorithm n times independently with the same data [10,11]. To pass the test, the test runs are expected to yield similar results with small variance. To do so, the reliability test of the three algorithms is carried out for the five benchmarks. Results are shown in the form of frequency chart in Fig. 5, Column (2). It is done in such a way that spin glass' minimum energy (E_{gs}) in ground state is counted and the probability to reach that state is also shown. The variance between the final energy states is given in Table 1. Experimental results from 100 trials indicate that the algorithm's final value has a small variance. In other words, final spin glass' energy at each trial is in the range of best responses. Even though the movement toward this final response is random in the above algorithms, they consistently reach the ground state.

9. Pareto Frontier

In Column (3), The Pareto front from the proposed composing algorithm can be seen. It shows the validity of energy reduction and avoiding local optimums for the five mentioned stock markets. The standard frontier for different λs is also drawn. For having a Pareto frontier, λ is considered in the range of 0.05–0.95 with 0.05 difference. For any λ, the spin glass' optimization state is found and its risk and capital return values are defined with points. The validity of the presented algorithm in finding optimization response with different λ is seen through comparing the resulting and standard (benchmark) Pareto fronts. Since the surface of whole optimization frontier is covered, one can conclude that the presented method gives adequate response for any λ. In paper [16], p. 1186, a comparison between Pareto frontier of some heuristic methods such as GA, TS, SA and NN is illustrated. Observe that these graphs confirm SG gives significantly better results than other mentioned heuristics.

10. Summary and conclusions

In this paper, a new combined method of local and global behaviors is presented. According to this algorithm, glass initially runs with local behavior, and then with changing conditions according to Theorem 1, the behavior is changed to global. This algorithm guarantees the accuracy and speed at the same time. Since Theorem 1 shows the upper limit for difference between local and global optimum of portfolio selection problem, the appropriate value for α, i.e. the appropriate time of changing glass behavior can be specified. Also as shown earlier, the composition behavior works better for larger problems (when N is large). A test of reliability and study of Pareto frontier confirm the theoretical analysis and the utility of composing behavior for faster convergence while reaching desirable solutions.

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