From Local Search to Global Conclusions: Migrating Spin Glass-Based Distributed Portfolio Selection

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***Abstract*—Spin glass optimization is a distributed technique inspired by the interactions in spin glasses in nature. Spin glasses are the lattices of spins where each spin is only a part of the entire solution, in contrast to genetic algorithms (GAs), where each chromosome represents a complete solution. The interaction between spins creates special optimal patterns given appropriate temperature. This optimization paradigm is promising in complex multiobjective optimization tasks because it allows high-computational parallelism among its member spins. Furthermore, since the overall network of spins represents only one solution, there is a great promise in computational efficiency when compared with other population-based/stochastic approaches such as GAs and simulated annealing. The nature of this method is also entirely different from other distributed frameworks such as Hopfield neural network since spins' paradigm of interaction does not have to be fully connected; i.e., the neighborhoods of interactions can expand or collapse, hence less computation and better convergence speed. In this paper, we apply a heuristic method based on the spin glass model that uses migration and elitism operators in addition to temperature control in order to trace out an efficient frontier in the optimization landscape. The proposed methodology is then applied to the problem of portfolio selection. Portfolio selection is one of the nondeterministic polynomial complete problems where each asset's behavior is similar to spin's behavior and it is therefore suitable as a case study. We show that, in proper circumstances, decrementing local energy of each spin can decrement global energy of the glass, and correspondingly, if the optimization problem can be suitably mapped to the glass, the expected cost function decreases.**

***Index Terms*—Ising spin glass model, local optimization, parallel processing, portfolio selection, simulated annealing.**

I. Introduction

QUALITATIVE and quantitative development of computer processors and memories has inspired the simulation of many complex systems at the macroscopic level based on the performance of atoms at the molecular level. This, together with the daily increasing human need to solve new problems, has led to further analysis of the characteristics and application of these types of models at macroscopic level with an emphasis on simulation. For instance, neural networks (NNs) inspired by the human brain, genetic algorithms (GAs) inspired by natural evolution, and computer antiviruses inspired by immune systems have emerged as new sciences. The Ising spin glass model is one of these newly emerged models in engineering disciplines [14], [15]. The main distinction is that the overall network of spins composes a complete solution, i.e., each spin is only a part of the solution; this is in contrast to other optimization heuristics, such as GAs, where each chromosome is a complete solution.

The Ising model is a network of spins in which spins interact magnetically, and consistently change their values to achieve a lower level of energy. When the system is in minimum energy (or temperature) state, there is no significant variation in spin values and the system has reached ground state (the state with minimum energy). This model has many characteristics among which are nonexponential growth of ground states with an increase in the number of spin bonds, effectiveness of environmental factors such as temperature on network behavior and ability to achieve the optimum state at variant temperatures. Many optimization problems can be solved according to these characteristics. In this paper, solving the portfolio selection problem has been targeted as a case study.

The problem of portfolio selection is one of the challenging problems in artificial intelligence due to the nondeterministic polynomial completeness of its search space. This problem also has many important and prevalent engineering applications in the industry, and in particular, in the area of finance and investment. In the portfolio selection problem, given a set of available assets, we want to find out the optimum way of investing a particular amount of money in the assets. Each one of the different ways to diversify the money between the several assets is called a portfolio.

For solving this portfolio selection problem, Markowitz [1], [2] presented the so-called mean-variance model, which assumes that the total return of a portfolio can be described using the mean return of the assets and the variance of return (risk) between these assets. The portfolios that offer the minimum risk for a given level of return form an efficient frontier. For every level of desired mean return, this efficient frontier indicates the best distribution of invested money.

Many intelligent and heuristic methods are used to solve this problem. Past activities in this area have focused on

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evolutional algorithms, GAs, particle swarms, simulated annealing (SA), NNs, and fuzzy-probabilistic methods. All these methods, though imperfect, lead to acceptable solutions. More specifically, Beasely, in 1998 [3], proposed an algorithm for finding a bound on the objective function by decomposing its original quadratic form into a sum of pseudobilinear functions. The decomposition to use is found via (approximate) solution of the dual of a Lagrangean decomposition of the initial problem. In 2000, Tanaka *et al.* [4] proposed two kinds of portfolio selection models based on fuzzy-probabilities and possibility distributions in Markowitz's model, in contrast to the conventional probability distributions. Based on a fuzzy-probability and a possibility distribution, portfolios are selected to minimize the variance of the return of a portfolio in a fuzzy-probabilistic model and the spread of the return of a portfolio in a possibilistic model, respectively. They claimed that the possibilistic approach is suitable due to the importance of expert judgment in data analysis. In 2003, Crama and Schyns [5] described the application of a simulated annealing approach to the solution of a complex portfolio selection model enriched by additional constraints. In 2004, Pafka *et al.* [6] and Yin *et al.* [7] described a model-based approach for a systematic investigation of the performance of various noise reduction procedures applied to portfolio selection and risk management. In 2006, Chen *et al.* [8] applied particle swarm optimization to the constrained portfolio selection problems which include transaction costs and taxes, the floor and ceiling constraints. In 2007, Lin and Liu [9] presented three possible models for portfolio selection problems with minimum transaction lots, and devised corresponding GAs to obtain the solutions. The results of their empirical study showed that the portfolios obtained using their proposed algorithm are very close to the efficient frontier, indicating that the proposed method can obtain near optimal and also practically feasible solutions to the portfolio selection problem in an acceptably short time. In 2007, Fernandez and Gomez proposed a heuristic method based on artificial NNs [10]. Their results were compared to those obtained using three other heuristic methods from the fields of GAs, Tabu search, and SA. They concluded that the NN model provides better solutions than the other three heuristic methods under certain conditions.

In this paper, as with the above methods, we try to solve the Markowitz portfolio selection problem according to its constraints, the point of departure being how we intend to solve it. The proposed distributed solution paradigm is based on a spin glass model. Local and global behaviors of the spins as well as the effect of migration and elitism on spin glass performance are considered. Finally, the results of the experiments are considered.

Following the introduction, the portfolio selection problem is described in Section II. The Ising spin glass model is discussed in Section III. A review of the literature related to the spin glass paradigm is given in Section IV. In Section V, portfolio selection problem is solved using an algorithm based on spin glasses and an analysis of its convergence is provided. Section VI illustrates the performance of this method on benchmark data. Finally, conclusions appear in Section VII.

II. Portfolio Selection Problem

Let us consider the Markowitz mean-variance model [1], [2] for the portfolio selection problem as stated below

$$\text{Min} \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \sigma_{ij} x_j \tag{1}$$

$$\text{Max} \sum_{i=1}^{N} \mu_i x_i \tag{2}$$

$$\text{subject to } \sum_{i=1}^{N} x_i = 1 \tag{3}$$

$$0 \le x_i \le 1, \qquad i = 1, \ldots, N \tag{4}$$

where N is the number of different assets, μ_i is the mean return of asset i, and σ_{ij} is the covariance between returns of assets i and j. The decision variable x_i represents the fraction of capital to be invested in asset i. Equations (1) and (2) are two cost functions that should be solved with constraints (3) and (4). μ_i is the mean return of asset i in n intervals of time, i.e., $\mu_i = \sum_{t=1}^{n} \frac{W_{ei}(t) - W_{bi}(t)}{W_{bi}(t)}$, where W_{bi} is the ith asset value at the beginning and W_{ei} is the ith asset value at the end of each interval.

Solving this problem with multiobjective optimization methods has been presented in [11]. A feasible solution of the portfolio selection problem is an optimal solution if there is no other feasible solution improving one objective without deteriorating the other. Usually, multiobjective optimization problems such as those in [12] have multiple nondominated optimal solutions. This set of solutions form what it is called an efficient frontier. For the problem defined in (1)–(4), the efficient frontier is an increasing curve that gives the best tradeoff between mean return and variance (risk).

In this paper we change the multiobjective problem into a multimodal problem with single objective function as follows:

Minimize

$$\lambda . \left[\sum_{i=1}^{N} \sum_{j=1}^{N} x_i \sigma_{ij} x_j \right] + (1 - \lambda) . \left[- \sum_{i=1}^{N} \mu_i x_i \right] \tag{5}$$

$$\text{subject to } \sum_{i=1}^{N} x_i = 1 \tag{6}$$

$$0 \le x_i \le 1, \ i = 1, \ldots, N. \tag{7}$$

In (5), let $\lambda \in [0, 1]$ be the risk aversion parameter. If $\lambda = 0$ then (5) represents maximum portfolio mean return (without considering the variance) and the optimal solution will be formed only by the asset with the greatest mean return. The case with $\lambda = 1$ represents minimizing the total variance associated with the portfolio (regardless of the mean returns) and the optimal solution will typically consist of several assets. Any value of λ inside the interval (0, 1) represents a tradeoff between mean return and variance, generating a solution between the two extremes, $\lambda = 0$ and 1.

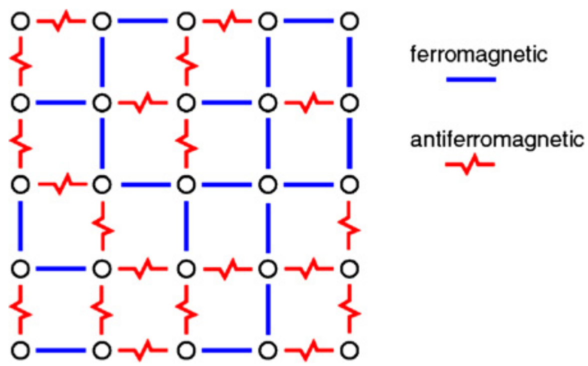

Fig. 1. 2-D spin glass with bond disorder. Spins are placed on the sites of a regular grid. They interact with their neighbors; the interaction is random, either ferromagnetic (straight lines) or antiferromagnetic (jagged lines) [14].

III. Spin Glass Model

Spin glass is a model which can be used to investigate the collective properties of physical systems made from a large number of simple elements. The important feature in this paradigm is that the interactions among these elementary components yield a collective phenomena, such as stable magnetization orientation and the crystalline state of metal or alloy. In the Ising spin glass model [13], an Ising spin on a lattice point takes on one of two possible values (directions) (i.e., ± 1 or up and down). By generalizing the Ising spin glass model to a XY spin glass model (hereafter referred to as a spin glass model for short) [14], [15], each spin can point to any direction in a plane instead of just two possible directions.

A suitable theoretical model describing spin glasses consists of N spins placed on the regular sites of a d-D lattice with linear extension L, e.g., quadratic ($N = L^2$) or cubic ($N = L^3$). The spins interact ferromagnetically or antiferromagnetically with their neighbors. A small example is shown in Fig. 1.

The energy of such a network comes from two contributions [24], [30] and can be written as follows:

$$E(\{x_i\}) = \left[-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{m} x_i J_{ij} x_j\right] + \left[-\sum_{i=1}^{N} h_i x_i\right] \qquad (8)$$

where $E(\{x_i\})$ is the energy of all spins, The sum i,j runs over all pairs of nearest neighbors, m is the number of nearest neighbors of each spin i that can be $m = 4$ in Van Neumann cellular automata (CA) or $m = 8$ in Moore CA [16] or $m = N$ for full connection. J_{ij} denotes the strength of the bond connecting spins i and j. $J_{ij} > 0$ describes a ferromagnetic interaction, while $J_{ij} < 0$ describes an antiferromagnetic interaction. The quantity h_i is the external field acting on spin i and describes the energy due to the spin's orientation. Also, the factor 1/2 corrects for double counting of the interaction between every two neighboring spins. Here, the task is to find a spin configuration x_i that minimizes the energy of the spin glass, given $\{J_{ij}\}$, $\{h_i\}$.

IV. Literature Review on Spin Glass Paradigm

There is also a wealth of existing literature on spin glasses in various domains in general, and physics in particular. For the sake of brevity as well as better focus, we will be concerned here with those research related to engineering, and in particular, optimization [14], [15], in which literature is relatively scarce. Minimum cost flow and matching problem are two examples of this kind. In Minimum cost flow problem, the ground state configuration of an Ising spin glass in a random environment, in which all energies are non-negative, can be obtained with Dijkstra's algorithm to find the shortest path in a directed network with non-negative cost on the edges. In the matching problem, the ground state of a 2-D spin glass model on a square lattice with nearest neighbor interaction with free boundaries can be mapped onto a matching problem of a general graph [15].

In 2001, Nishimori [17], [18] considered the application of spin glasses in transferring information in noisy channels. He stated that many aspects of the concepts related to spin glasses can be applied to transferring information in noisy channels, because most of the available methods in this application area have probabilistic structure that can conveniently correspond to spin glasses. Also in 2001, Sourlas [19] showed a deep relation between error-correction codes and certain mathematical models of spin glass. In particular, minimum error probability decoding is equivalent to finding the ground state of the corresponding spin system. In 2004, Horiguchi *et al.* [20] proposed a spin glass-based routing algorithm for adaptive computer networks, in such a way that a spin is allocated to every node in the network and packets are routed according to the minimum energy of each node.

In 1996, Bennett and Shapiro [21] presented a simple GA consisting of selection, mutation and crossover which is searching for the ground states of simple random Ising spin systems and spin glass. Gallucio *et al.* [22], in 1998, were first to indicate the suitability of the spin glass paradigm for portfolio selection problem. Later in 1999, Gabor and Kondor [23] used spin glasses for the first time in solving the portfolio selection problem with regard to its constraints. In their paper, they used a similar energy function to that of a Hopfield NN [10]. They showed good performance for low risk assets, but their algorithm deteriorated for high-risk assets. Furthermore, their algorithm had slow convergence since the neighborhood of each spin spanned the complete spin network. This slow convergence was more pronounced with increasing number of spins. To the authors' best knowledge, the current work is the next research.

V. Solving Portfolio Selection Problem Using Spin Glass Paradigm

Before proceeding to the solution, we should deal with the question of why we are using spin glass in solving portfolio selection problem. The answer to this question can be found in [22]. In their paper, Galucchio *et al.*, stated that, for many systems, an increase in the number of state interactions can yield to an exponential increase of the number of optimal answers (state with minimum value). Most available methods do not have the capability of searching in the great repertoire of states and are not effective in the desired time period. But if the problem can be properly mapped into a spin glass network,

Algorithm 1: Global Spin Glass Optimization
Begin
1 *Select one spin i randomly at a time*
2 *Change the state of spin i by* ε *(very small change) and change all the neighborhood spins (When* $m = N$*, all the spins are neighborhoods of each spin) to satisfy constraints (6) and (7).*
3 *Calculate the new energy of the changed spin and its neighbor spins globally* $(m = N)$.
$E_{\text{new}} = \sum_{i=1}^{m} E_i$
4 $\Delta E = E_{\text{new}} - E_{\text{old}}$
5 *If* $\Delta E < 0$ *then accept this change else*
6 *If* $\Delta E > 0$ *then accept this change with probability* $e^{-(\Delta E/kT)}$
7 *Continue this process with decreasing temperature till either* ΔE *remains near* 0 *for several iterations [i.e., the system has reached steady state, or T has reached near* 0 *([system has been cooled)]*
End

the increase in the number of spin bonds does not increase the number of ground states exponentially. This characteristic helps spin glasses to become an effective paradigm for solving optimization problems with many internal interactions such as in the portfolio selection problem.

To present our method, we initially map the portfolio selection problem into a spin glass computational model and then find its ground state by looking at the objective function, (5), of the portfolio selection problem and comparing it with the spins energy function (8) of the spin glass model. We obtain the values for the interaction strength as follows:

$$J_{ij} = -2\lambda\sigma_{ij} \tag{9}$$

$$h_i = (1-\lambda)\mu_i. \tag{10}$$

The decision variable x_i represents the proportion of capital to be invested in asset i; and in spin glass, we can define x_i to be the state of spin i. So the problem of portfolio selection can be solved by minimizing the mapped function as in (8).

A. Proposed Spin Glass Algorithm

For finding the minimum of optimization function (5) with regard to constraints (6) and (7), we first randomly place the possible assets into a $L_1 \times L_2$ lattice-like structure such that $N = L_1 \times L_2$, where N is the number of assets. All of the spins in this structure are initialized to $1/N$. Therefore, we can select the best assets using Algorithm 1.

In Algorithm 1, E_{old} and E_{new} represent the total energy of the network before and after applying changes. T is the temperature of the network during applying changes. ε is a small value, here 0.05, that shows the change of spin state in each spin flip. Our analysis indicates that ε is inversely proportional to $\overline{J}$(mean value of J_{ij}), particularly when $\overline{J}$ is large. For smaller $\overline{J}$, the value of ε becomes less important since the spin movement has less global effect. Overall, our experiments indicate that a value of 0.05 is a good value across all benchmarks. Also the temperature of the network is initially very high in order to enable diverse search.

According to the algorithm, a spin is chosen randomly at every flip and the value of the selected spin is increased by ε. Accordingly, the value of neighboring spins changes to meet the constraints (6) and (7). The amount of energy is then estimated. If there is a decrease, the change is accepted; otherwise it is only accepted with probability $e^{-(\Delta E/kT)}$ with $k = 1$ [24]. This procedure continues until either the minimum energy is achieved or system is completely cooled.

For the heating and cooling schedule, procedures related to SA are used, as in [25]. To do so, the temperature of the network is considered to be initially set to $T_0 = 1$ (at high temperatures all states can occur). Each time the changes are applied, the temperature gradually decreases until it reaches near zero. Temperature variations can be governed by the following formula:

$$T(n) = \frac{T_0}{n^2}, \qquad n \geq 1 \tag{11}$$

where n represents the number of epochs. The stop condition of algorithm is the iteration of single result in number of defined steps continually with regard to defined precision. For example, all experiments' results have been measured by ten same results with a precision of 10^{-7}.

B. Constraint Satisfaction

In portfolio selection problem, two limitations (6) and (7) have brought about some considerations. In order to satisfy the constraint (6), i.e., to keep the total amount of spins constant at 1, when any spin is increased by ε ($x_i := x_i + \varepsilon$), ε/m is subtracted from the spin's neighbors ($x_j := x_j - (\varepsilon/m)$ where $j = 1, ..., m$). If $x_i \geq 1$ then $x_i := 1$ and the extra value is subtracted from ε; alternatively, if for each neighbor $x_j - (\varepsilon/m) \leq 0$ then $x_j := 0$ and the value difference is added to x_i. According to the above explanation constraint (7) is also satisfied.

C. Algorithm Analysis

One may ask: what is the role of ε and the probability $e^{-(\Delta E/kT)}$ in the network and why does the algorithm converge to the optimal solution?

The Metropolis algorithm [26] defines the probability of state transition from $\{x\}$ to $\{x'\}$ as

$$P\left(\{x\} \rightarrow \{x'\}\right) = e^{-(\Delta E/kT)}. \tag{12}$$

Because the energy function is continuous, if ε selected is sufficiently small, ΔE is small. Therefore, changing T has a central role in control of the probability and it is appropriate to using SA in our approach. From (12), when T is large (i.e., high temperature), spin has a higher probability to change to a state which will increase total system energy than when T is a small value (low temperature). So in high temperature, the system tends to escape from local minima, and in low temperature, the system tends to converge to a global minimum (ground state). This enables the system to avoid a local minimum and to seek a global minimum.

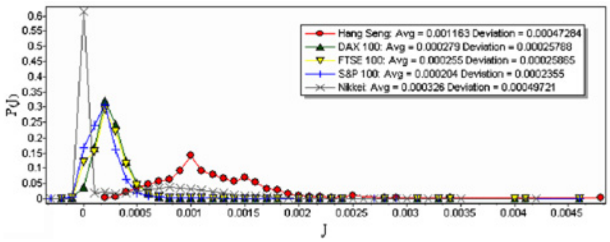

Fig. 2. Probability density functions for covariances between assets in five major stock markets from 1992 to 1997 from data in [27].

VI. Experimental Results

In order to verify the effectiveness of the above algorithm, the benchmarked "standard efficient frontier" (Pareto Front) is compared with the efficient frontier resulting from the proposed method. Also spin glass behavior is analyzed under different temperatures as well as multiple architectures of interaction.

Experiments on the benchmark data were originally performed in [27]. These benchmark data are presented in text file format as follows.

Number of assets (N); and for each asset i ($i = 1, \ldots, N$): *mean return* as well as *standard deviation of return*; for all possible pairs of assets: i, j, *correlation* between asset i and asset j. The above data were taken from five major stock exchange markets, during the time period extending from March 1992 to September 1997. These five stock exchange markets include Hang Seng in Hong Kong (31 assets), Deutscher Aktien Index (Dax 100) in Germany (85 assets), Financial Times London Stock Exchange (FTSE100) in Britain (89 assets), Standard and Poor's (S&P 100) in USA (98 assets), and Nikkei in Japan (225 assets). Probability density function (pdf) of covariance $P(J)$ of each stock market for the given data has been shown in Fig. 2.

As can be observed, the $P(J)$ of the five given stock markets have small mean and variance. Standard efficient frontier for each of these five stock markets in the available time period is characterized by mean return as in (2) and variance of return as in (1). Fig. 3 illustrates these efficient frontiers.

Various tests concerning the analysis of spin glass behavior are considered here as follows.

1) Analysis of spin glass convergence with $m = N$ and $m < N$ based on local or global behavior of the spin glass network and assessing the possibility of achieving the optimum solution.
2) Examining the efficient frontier resulting from executing the algorithm and comparing it with the standard efficient frontier [27].
3) Analysis of the rate of convergence for the network.
4) Analysis of spin glass behavior in case of using migration and elitism.
5) Analysis of parallel processing in spin glass network.

All of the experiments were performed using Borland Delphi 6.0 running on a Pentium 2.4 GHz PC, under Windows XP operating system. It should be mentioned that each spin flip equals performing the algorithm once and each epoch equals 100 flips.

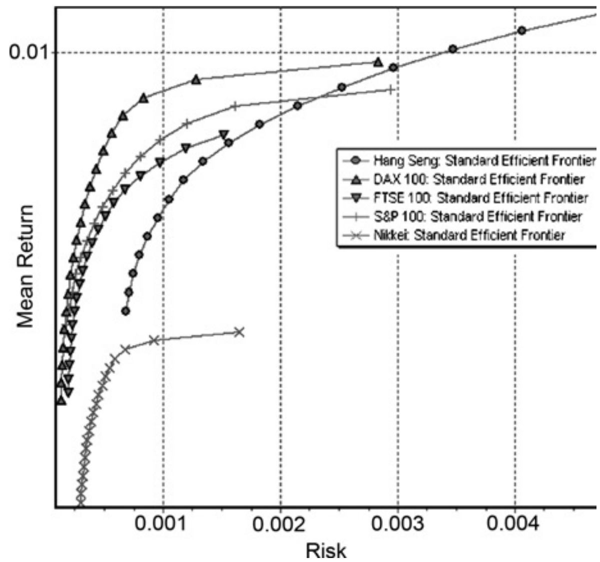

Fig. 3. Standard efficient frontier for benchmark data from five major stock markets [27].

A. Analysis of Spin Glass Behavior

Equation (8) (the spin glass energy function) corresponds to (5) (the actual cost function of the portfolio selection problem). From this mapping, we can analyze the portfolio selection problem using the spin glass model. There are two types of behaviors that can be considered for spin glasses.

1) *Global Behavior (G):* In this behavior, due to spin–spin interactions, variation in the value of one of the spins affects all the spins and consequently the total energy. In this case, an increase or decrease in the spin energy yields a similar change in the overall energy of the network. While this behavior is desirable, it imposes quadratic time complexity since the number of interactions (bonds) grows with the size of the glass.
2) *Local Behavior (L):* In this behavior, if there is a variation in the value of a spin due to the forces between the spin and its neighboring spins, this variation is only propagated to the spin's neighbors. In this case, the energy variation of the spin and its neighbors would not necessarily correspond to the global energy changes. While global conclusions cannot be made with this type of local behavior, its computational complexity remains constant.

The following experiments are performed in order to investigate the above two behaviors.

1) *Spin Glass Behavior Analysis:* The results of the proposed algorithm performed on the five stock markets are shown in Fig. 4. Each row presents spin glass behavior for each market, first column in this figure presents the state of spin glass energy in each epoch, second column presents spin glass local behavior using operators such as elitism and migration. In solving the portfolio selection problem, four different behaviors, each with certain characteristics, appear as follows.

a) *Global behavior with $m = N$:* First column presents the spin glass behavior when $m = N$. In this state, cost function equals the overall spin glass energy function. In Section V-C, it is shown that spin glass tries to find states with lower energy (toward ground states), therefore in this state, optimum portfolio selection equals optimum spin glass states. The advantage of this behavior is its accuracy in obtaining the optimum portfolio, but it is slow for large N.

b) *Local behavior with $m = N$:* This behavior is concerned with the local energy of each spin when $m = N$. In Fig. 4, column 1, the global and local behavior of all neighbors are compared. Since all of the spins in the glass are neighbors of a given spin, local behavior is similar to global behavior. However, due to the decrease in local energy, the rate of convergence can be slower than global behavior.

c) *Global behavior with $m < N$:* Under such a condition, the total energy variation of the network is calculated based on the variation in each spin, but the variation in each spin causes a variation only in its neighbors and not all of the spins. Therefore, in this state the optimum portfolio does not necessarily correspond to the optimum spin glass state. However, according to the experiment shown in column 2, Fig. 4, we can find a good approximation for optimum portfolio by applying small variations in the spin glass structure. The migration and elitism operators modify the placement of spins in a way to enable exploiting the glass structure to find the optimum portfolio. These two migration/elitism operators are discussed in Section VI-C.

d) *Local behavior with $m < N$:* In this state, neighbors of an altered spin are adjusted, but this variation is not further propagated to all spins in the glass. This behavior is the most adhereable to parallel processing and is quite fast; but it also has two difficulties. Firstly, $m < N$ causes a deviation in the value of cost function and spin glass energy (energy level difference), and accordingly, a decrease in one may not be accompanied by a decrease in the other. Secondly, the local activity of spins does not necessarily lead to global optimization. Therefore, analyzing the given asset data and changing the spin glass structure accordingly are among the proposed options to overcome these two problems.

In following the above discussion, Table I shows the cost functions at ground states and computing time for the above four behaviors. Local behavior is computed with neighborhood size of 8. When $m = N$, local behavior and global behaviors reach same ground sates, i.e., the actual cost function. When $m < N$, a deterioration in performance but better computing time is gradually observed.

After analyzing the portfolio data presented in Fig. 2, we find that maximum covariance between pairs of assets that equals minimum value of J [according to (9)] is less than 0.05, therefore the energy difference between local and global behaviors is insignificant, and the parallel processing characteristic of spin glasses is used in this algorithm. This analysis is also confirmed by considering Table I where the glass reaches similar final ground states with $m = N$ and $m = 8$. The changes in Algorithm 2 are then made in Algorithm 1.

Algorithm 2: Local Spin Glass Optimization
Begin
1 *Select one spin i randomly at a time*
2 *Change the state of spin i by ε (very small change) and change all the neighborhoods state to satisfy constraints (6) and (7).*
3 *Calculate the local energy of the changed spin and its neighborhood spins* ($E_{\text{new}} = \sum_{i=1}^{m} E_i$)
4 $\Delta E = E_{\text{new}} - E_{\text{old}}$
5 *If* $\Delta E < 0$ *then accept this change else*
6 *If* $\Delta E > 0$ *then accept this change with probability* $e^{-(\Delta E/kT)}$*else*
7 *If reject then change spin location with one of its neighbors. (For migration, exchange spin with another randomly chosen spin; for elitism, If spin has higher energy than any of its left or upper-left neighbors, they exchange places).*
8 *Continue this process with decreasing temperature till either* ΔE *remains near* 0 *for several iterations (i.e., the system has reached steady state, or* T *has reached near* 0 *(system has been cooled)*
End

In Algorithm 2, E_{old} and E_{new} are the energy of each spin plus the energy of its neighbors before and after applying changes. The results of running Algorithm 2 are shown in Fig. 4, for $m < N$ and $m = N$ with local and global search. Also, the probability of reaching ground state (p) is shown in column 1. When $m = N$, p is equal to 1. However, when $m < N$ (without using migration and elitism operators), p is often disappointingly far from 1. Whereas, in column 2, when $m < N$ (with using migration and elitism operators), p is near to 1 and spin glass is near its ground state.

2) *Comparison with the Standard Efficient Frontier:* In this experiment, the efficient frontier for benchmark data is plotted using the proposed algorithm, as illustrated in Fig. 5. In order to accurately determine the efficient frontier, it has been plotted for each stock market with a range of λ starting from 0.05 to 0.95, with incremental steps of 0.05. For each λ, Algorithm 2 is performed with 100 epochs, and its risk and return has been marked with points. The results are presented in Fig. 5. As can be observed, for all stock markets the result of running the algorithm is consistent with the efficient frontier and this supports the efficiency of the spin glass in this regard.

B. Study of Different Dimensions and Neighborhood Size

This experiment analyzes the effect of dimensions on spin glass behavior. We consider a network with different sizes for Nikkei stock market. Fig. 6 shows the results of the following three network sizes: 1 × 225, 5 × 45, 15 × 15. For $m = N$, the structure of the network has no effect on the rate of convergence, because all of the spins are connected to one another; but this is different for $m < N$. As illustrated in Fig. 6, the more connections created by spin

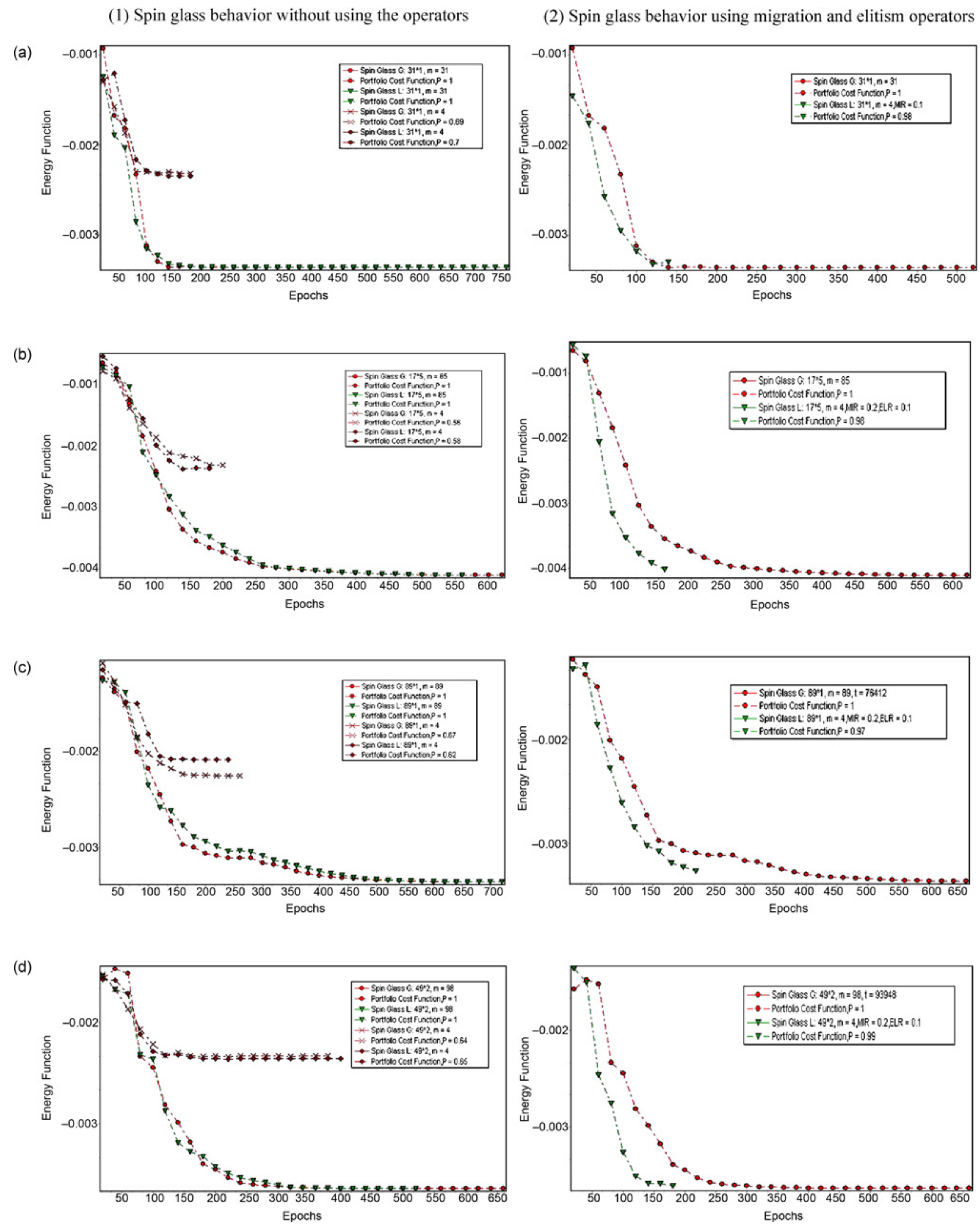

Fig. 4. Comparison between local and global behaviors, with or without migration/elitism operators. Column 1: spin glass size, number of neighbors, and probability of reaching ground state (p). Column 2: migration and elitism rates are presented. (a) Hang Seng. (b) Dax 100. (c) FTSE 100. (d) S&P 100.

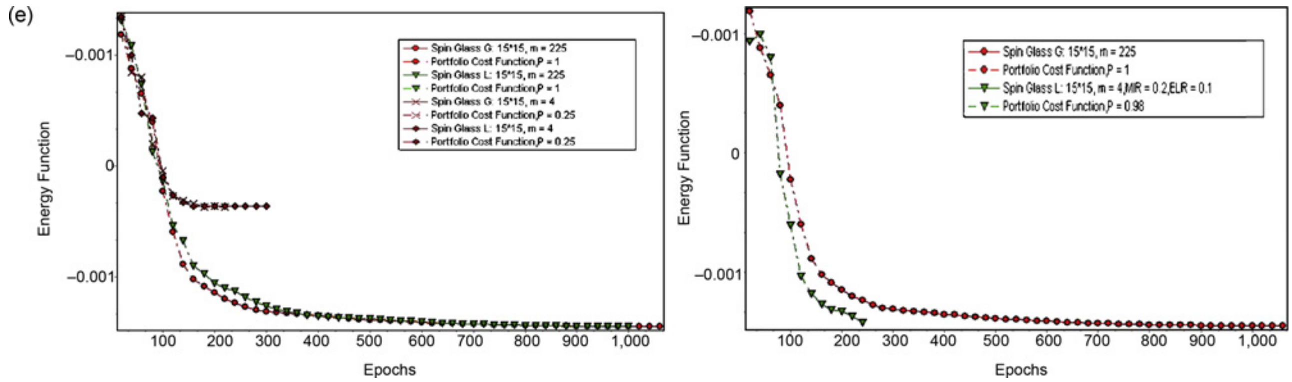

Fig. 4. (Continued) (e) Nikkei.

TABLE I

COMPARISON BETWEEN SPIN GLASS BEHAVIORS FOR VARIOUS NEIGHBORHOOD SIZES FOR DIFFERENT STOCK MARKETS

	Actual Cost Function	Global Behavior with $m = N$		Local Behavior with $m = N$		Global Behavior with $m = 8$		Local Behavior with $m = 8$	
		Cost Function	Time (ms)	Cost Function	Time (ms)	Cost Function	Time (ms)	Cost Function	Time (ms)
Hang Seng ($N = 31$)	−0.0034	−0.0034	4626	−0.0034	3470	−0.0033	3210	−0.0029	2180
DAX 100 ($N = 85$)	−0.0041	−0.0041	65 268	−0.0041	51 847	−0.0039	20 735	−0.0038	17 659
FTSE ($N = 89$)	−0.0034	−0.0034	75 631	−0.0034	61 329	−0.0032	41 003	−0.0029	31 981
S&P ($N = 98$)	−0.0036	−0.0036	81 231	−0.0036	62 431	−0.0036	46 164	−0.0031	32 039
Nikkei ($N = 225$)	−0.0014	−0.0014	418 140	−0.0014	291 200	−0.0013	121 320	−0.0012	81 321

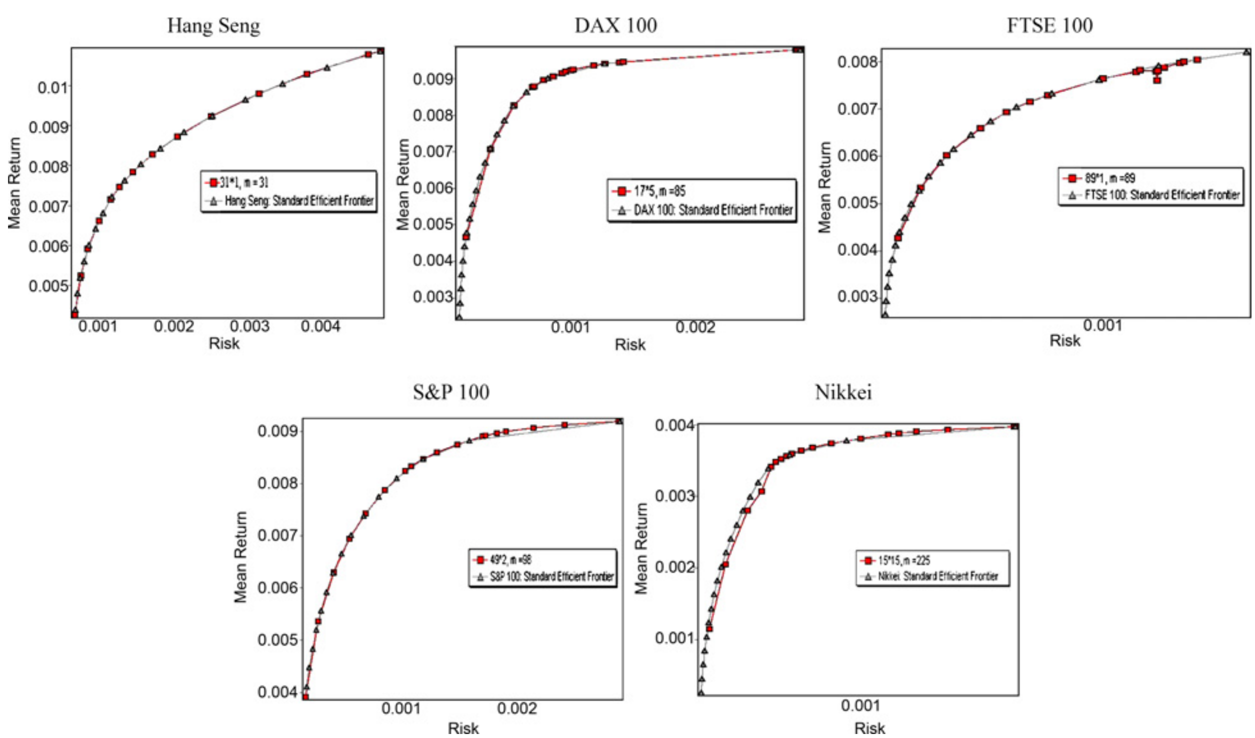

Fig. 5. Efficient frontier obtained from the algorithm compared to standard efficient frontier from benchmark data [27].

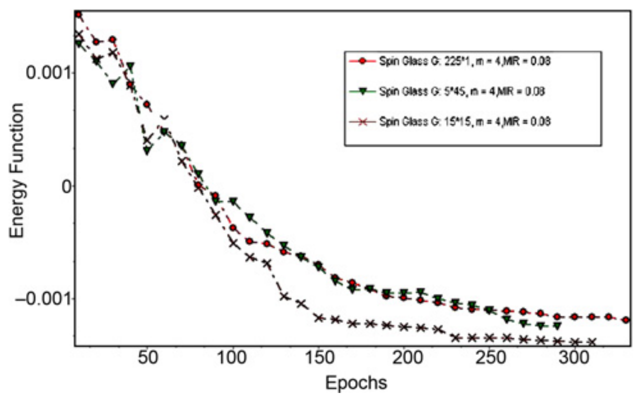

Fig. 6. Rate of convergence for 15 × 15 spin glass is better than 1 × 225 and 45 × 5, and behaves more efficiently.

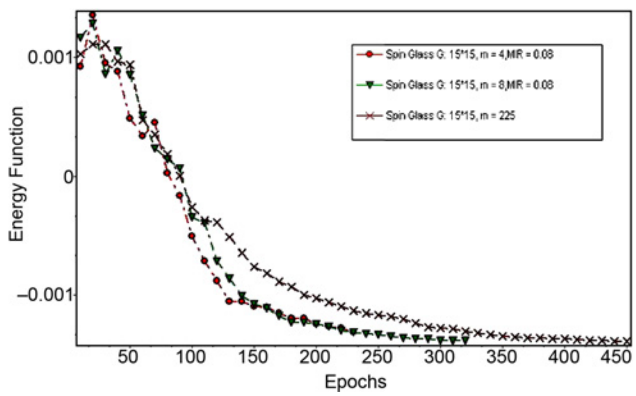

Fig. 7. Accuracy for 15 × 15 spin glass with $m = 4$, $m = 8$, and $m = 225$.

glass exist, the higher the rate of convergence and the better the performance is. Therefore, a spin glass structure with equal dimensions is the most efficient structure (has the highest rate of convergence) in solving the problem of optimization.

In another experiment, we considered increasing the number of connections between spins in the 15 × 15 structure. This experiment was conducted with $m = 4$, $m = 8$ and $m = N = 225$.

As illustrated in Fig. 7, when $m = 225$, accuracy is at its highest. Similarly, the algorithm is more accurate when $m = 8$ than when $m = 4$.

C. Migration and Elitism Operators

As mentioned before, one of the weaknesses of Hopfield NN [10], SA [5], spin glass [28], [29] as well as the proposed algorithm (when $m = N$) is the slow rate of convergence. In Section VI-A, it is shown that due to low covariance in portfolio data, we can use spin glass with local behavior to obtain the optimum solution without a significant loss of accuracy. Therefore, we suggest using GA operators such as migration and elitism in order to increase the rate of convergence. In this paper, migration refers to the transferring of selected spin to another random location which is done with the rate *MIR*. Elitism gradually and locally moves spins with higher spin level s_i toward each other and toward the upper left corner of the glass. This is a local operation by which

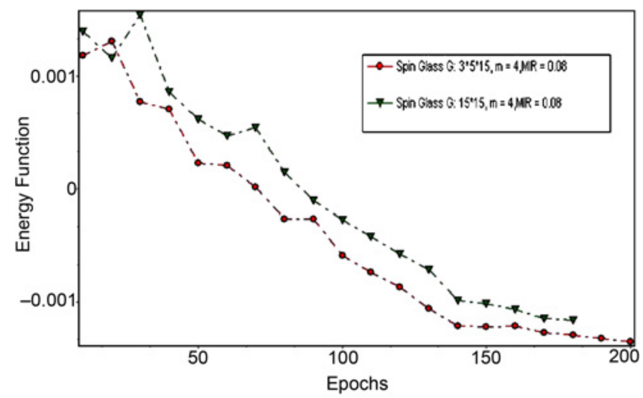

Fig. 8. Increased speed of convergence in parallel processing: three separate 5 × 15 spin glasses compared to one 15 × 15 spin glass.

TABLE II
EFFICIENCY OF PARALLEL SPIN GLASS EXECUTION

Spin Glass Behavior	No. of Processors	Mean Execution Time (ms)
Hang Seng ($N = 31$)	1	893
	2	508
	3	853
DAX 100 ($N = 85$)	1	4069
	2	2666
	3	2386
FTSE ($N = 89$)	1	4017
	2	2885
	3	2682
S&P 100 ($N = 98$)	1	4840
	2	3072
	3	2535
Nikkei ($N = 225$)	1	23 309
	2	17 738
	3	10 482
	4	9898

spins compare their own energy with that of their upper and upper-left neighbors. If it has higher spin level than any of the above two neighbors, they exchange places. This is done with an elitism rate of *ELR* for the selected spins. The replacement of spins by the elitism operator causes the elite spins to be placed next to each other and this causes a competition among the elite spins and hence increases the rate of convergence significantly. Column 2 in Fig. 4 shows the significant increase in rate of convergence [probability of reaching ground state (p)] when using these two operators.

D. Parallel Processing

Local search and suitability for parallel execution in spin glass network is another feature of the proposed approach. If spins are categorized, arranged in separate processors, and executed in parallel, the rate of convergence can be significantly increased. The following experiment, on a 15 × 15 network, analyzes the parallel processing characteristics of this algorithm. Here, the 15 × 15 network is divided into three 5 × 15 networks, where the networks interact only through their boundary spins. If each of the three networks is simultaneously executed by a separate processor, there will be a significant increase in the speed of convergence (since the authors of the paper did not have access to multiprocessor computers, multiprogramming technique is used here). At each flip, each processor chooses a spin and runs the algorithm; therefore at each flip, three spins (one by each processor) are chosen and optimized. Fig. 8 shows the results of running the above network by the three processors.

The efficiency of parallel execution is shown in Table II. Execution time for each program is compared for each of the five stock markets. For instance, the execution time of finding ground state of spin glass for Nikkei stocks for four parallel processors versus one processor is 9898 ms versus 23 309 ms. Yet the number of parallel processors cannot be gainfully used more than a limited number because of the overhead, communication and switching time between parallel processors. For example, the Hang Seng stock market shows a longer execution time for the three parallel processors (853 ms) than two parallel processors (508 ms).

VII. Summary and Conclusion

In this paper, a new spin glass-based optimization algorithm has been introduced that can solve the portfolio selection problem. In order to the prove the efficiency of the algorithm, the efficient frontier from this algorithm was compared with the standard efficient frontier mentioned in benchmark data of five major stock markets. It was found that the proposed method is superior to other benchmark methods such as Hopfield NN and SA, concerning its accuracy. However, the algorithm is computationally intensive. In order to increase its speed, input data was analyzed. It was shown that benchmark data for the five major stock markets have small covariance and, therefore, the problem can be suitably solved by local search. Therefore, the algorithm was modified to include local search such that global behavior can emerge. Experiments showed that the use of local search can significantly increase the speed of the algorithm. However, this increased speed leads to a decrease in accuracy. The elitism and migration operators were then introduced to address this problem and to increase the accuracy while a local search was performed. Finally, the resulting architecture was shown to be amenable to parallel processing.

In summary, analysis of the results yields the following general conclusions. First, the proposed paradigm aims to achieve global optimization by parallel local search. Experiments show that the covariance among assets, rather than their return, has a more significant effect on suitability of local search. The smaller the covariance between pairs of assets, the more efficient the local search will be. Second, the movement of the spins (assets) is of great importance in local search. According to this algorithm, movement due to elitism as well as random migration, both increase the speed of convergence. This could explain the success of algorithms based on particle swarms where particle movement is of great importance. Third, unlike many competing algorithms that are sensitive to the value of λ, the proposed algorithm can find the efficient frontier for any λ. Fourth, the increased speed from local search as well as maintaining global search performance due to the novel migration and elitism operators can help solve many optimization problems.

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