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Extremal optimization vs. learning automata: Strategies for spin selection in portfolio selection problems

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ABSTRACT

Nowadays, various imitations of natural processes are used to solve challenging optimization problems faster and more accurately. Spin glass based optimization, specifically, has shown strong local search capability and parallel processing. But, spin glasses have a low rate of convergence since they use Monte Carlo simulation techniques such as simulated annealing (SA). Here, we propose two algorithms that combine the long range effect in spin glasses with extremal optimization (EO-SA) and learning automata (LA-SA). Instead of arbitrarily flipping spins at each step, these two strategies aim to *choose* the next spin and selectively exploiting the optimization landscape. As shown in this paper, this selection strategy can lead to faster rate of convergence and improved performance. The resulting two algorithms are then used to solve portfolio selection problem that is a non-polynomial (NP) complete problem. Comparison of test results indicates that the two algorithms, while being very different in strategy, provide similar performance and reach comparable probability distributions for spin selection. Furthermore, experiments show there is no difference in speed of LA-SA or EO-SA for glasses with fewer spins, but EO-SA responds much better than LA-SA for large glasses. This is confirmed by tests results of five of the world's major stock markets. In the last, the convergence speed is compared to other heuristic methods such as Neural Network (NN), Tabu Search (TS), and Genetic Algorithm (GA) to approve the truthfulness of proposed methods.

1. Introduction

Similar to artificial neural networks, genetic algorithms, and ant colony systems, spin glass is a paradigm that is inspired from the governing laws of nature. However, as compared to many of its older counterparts, the main distinguishing feature of spin glasses is their unique distributed parameter optimization by emphasizing strong parameter interaction. More specifically, spin glass model is a system of spins interacting with each other due to the existence of magnetic property among them. These spins change their state frequently to reach a lower energy level. When the system is at its minimum energy (or minimum temperature) state, there is no longer a visible change in spins' states and the system is said to have reached its ground state [1].

In contrast to most other optimization algorithms such as GA (Genetic Algorithm) where each chromosome represents a complete solution, every spin is only a *part* of an entire solution. The complete solution is found by the interaction of the many spins in the glass. The spin glass paradigm is therefore a promising paradigm of adaptive distributed systems. In addition, the spin glass model enjoys lots of properties including limited interaction of each spin with neighboring spins [2], non-exponential growth of optimized (ground) states with the increase in spin glasses' number of bonds [3], the effectiveness of environmental factors such as temperature on system behavior, and a continuing movement toward optimized states at different temperatures [4].

Considering these capabilities, many optimization problems can be solved using such distributed facility [2]. But like many other heuristic methods, the rate of convergence of finding ground states is low when the problem dimension grows [5]. More specifically, this is reported to be a challenging aspect of the more conventional approaches such as the simulated annealing (SA) as reported earlier in [5,6].

To speed up the spin glass's rate of convergence, it would be desirable to choose the "right" spin that promises most improvement in terms of convergence rate and accuracy. This would be in contrast to the standard approach where spins are chosen arbitrarily. In this paper, we address this problem by combining SA with

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two other local search strategies, specifically, learning automata (LA) [7] and extremal optimization (EO) [8].

In contrast to SA, learning automata allows the superior spins to be selected more often based on their past ability to improve the glass. This can potentially lead to increased rate of convergence once the superior spins are found. But, it is not clear how many "experiences" are needed to determine a superior spin, and if the superior spin remains superior long enough for this strategy to be gainfully used. Also, this increased speed could be at the expense of reaching a local optimum due to the resulting selective search.

In EO, on the other hand, the spin with lowest energy is chosen to change its state with a higher probability. In contrast to LA, this scheme works since changing each spin influences its neighboring spins, and so they also change. If the total changes lead to a reduction in glass energy, the overall state of the glass improves and the correlation between spins increases. Hence, any change in each spin's state would lead to rearrange major parts of the glass. In 2001, Boettcher and Percus likened this property to an avalanche that can lead to a faster survey of different spin glasses' states and an increased rate of convergence [8]. The most obvious difference between EO algorithm and such other exploiting algorithms such as the Genetic Algorithm is the algorithm need to know the spin's fitness (local fitness) in addition to global fitness of solution. So, the higher speed to find the ground state without losing accuracy is yielded.

Here, we investigate two new hybrid methods based on extremal optimization (EO-SA) and learning automata (LA-SA) and compare them with SA [6] on a Markowitz portfolio selection problem. Section 2 reviews the various applications of spin glasses in solving optimization problems. Section 3 provides a mathematical description of spin glasses. The portfolio selection problem is discussed in Section 4. This section also explains how this problem can be mapped onto a spin glass. The three algorithms, SA, LA-SA and EO-SA, are then presented in Section 5. In Section 6, the experimental results from applying the above algorithms to five of the world's reputable stock markets are provided. The reliability test of algorithms and their performance validity is studied in Section 7. In Section 8, the resultant Pareto frontier is compared against the benchmark Pareto frontier. In Section 9, phase transition analysis of the three algorithms EO-SA over SA and LA-SA are presented and finally in Section 10, the convergence speed is compared to other heuristic methods such as Neural Network (NN), Tabu Search (TS), and Genetic Algorithm (GA) to approve the truthfulness of proposed methods.

2. Literature review on spin glass paradigm

There is a wealth of existing literature on spin glasses in various domains in general, and physics in particular. For the sake of brevity as well as better focus, we are concerned here with those research related to engineering, and in particular, optimization, in which literature is relatively scarce. Minimum cost flow and matching problem are two examples of this kind [10]. In minimum cost flow problem, the ground state configuration of an Ising spin glass in a random environment, in which all energies are non-negative, can be obtained with Dijkstra's algorithm to find the shortest path in directed network with non-negative cost on the edges. In the Matching Problem, the ground state of a two dimensional spin glass model on a square lattice with nearest neighbor interaction with free boundaries can be mapped onto a matching problem of a general graph [2,10,11].

In 1999, Gabor and Kondor [15] used spin glasses for the first time in solving the portfolio selection problem with regard to its constraints. In their paper, they used a similar energy function to that of a Hopfield neural network [16]. In 2001, Nishimori [12,13] considered the application of spin glasses in transferring information in noisy channels. They stated that many aspects of the concepts related to spin glasses can be applied to transferring information in noisy channels, because most of the available methods in this application area have a probabilistic structure that can conveniently correspond to spin glasses. In 2004, Horiguchi et al. [14] proposed a spin glass-based routing algorithm for adaptive computer networks, in such a way that a spin is allocated to every node in the network and packets are routed according to the minimum energy of each node. Later in 2009, Vafaei Jahan and Akbarzadeh in [6] introduced migration and elitism operators to find ground state of spin glasses with only a limited number of bonds, i.e. *short range* spin glasses. There [6], authors exploited *local* interaction among spins. In contrast, we consider here the *long range* effect of spin interaction by investigating the use of extremal optimization (EO) and learning automata (LA).

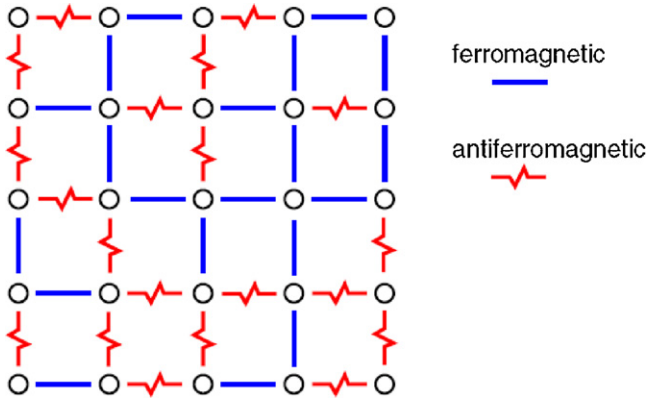

Fig. 1. A two-dimensional spin glass with bond disorder. Spins are placed on the sites of a regular grid. They interact with their neighbors, the interaction is random, either ferromagnetic (straight lines) or antiferromagnetic (jagged lines) [2]. Spin glass tries to find the ground state (configuration with minimum energy in Eq. (1)) by changing the spins' states.

The EO heuristic was first motivated by the Bak–Sneppen model of biological evolution [9] in 1993 for a lattice (glass) of cooperating species (spins). Some applications of this method were analyzed in solving optimization problems, including, solving the problem of travel salesman problem [11], graph partitioning [23,24], and graph coloring [25,26]. Since there are relatively few solved problems by this method, it is expected that the importance of this method be studied more in the future [28].

3. Spin glass model

Spin glass is a model which can be used to investigate the collective properties of physical systems made from a large number of simple elements. The important feature in this paradigm is that the interactions among these elementary components yield a collective phenomenon, such as stable magnetization orientation and the crystalline state of metal or alloy. In the Ising spin glass model [1,17], an Ising spin on a lattice point takes on one of two possible values (directions) (i.e. ± 1 or up and down). By generalizing the Ising spin glass model to a XY spin glass model (hereafter referred to as spin glass model for short) [2,11], each spin can point to any direction in a plane instead of just two possible directions.

A suitable theoretical model describing spin glasses consists of N spins placed on the regular sites of a d-dimensional lattice with linear extension L, e.g., quadratic ($N = L^2$) or cubic ($N = L^3$). The spins interact ferromagnetically or antiferromagnetically with their neighbors. A small example is shown in Fig. 1.

The energy of such network comes from two contributions [4,17] and can be written as below:

$$E(\{x_i\}) = \left[-\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{m} x_i J_{ij} x_j\right] + \left[-\sum_{i=1}^{N} h_i x_i\right] \quad (1)$$

where $E(\{x_i\})$ is the energy of all spins; the sum i, j runs over all pairs of nearest neighbors; m is the number of nearest neighbors of each spin i (that can be $m = 4$ in Van Neumann cellular automata (CA), or $m = 8$ in Moore CA [18], or $m = N$ for full connection); in this paper, we proposed our method on $m = N$ (long range spin glass). In this behavior, due to spin-spin interactions, variation in the value of one of the spins affects all the spins and consequently the total energy. When $m < N$ (for short range spin glass), if there is a variation in the value of a spin due to the forces between the spin and its neighboring spins, this variation is only propagated to the spin's neighbors.

J_{ij} denotes the strength of the bond connecting spins i and j. $J_{ij} > 0$ describes a ferromagnetic interaction, while $J_{ij} < 0$ describes an antiferromagnetic interaction. The quantity h_i is the external field acting on spin i and describes the energy due to the spin's orientation. Also, the factor 1/2 corrects for double counting of the interaction between every two neighboring spins. Here the task is to find a spin configuration x_i that minimizes the energy of the spin glass, given $\{J_{ij}\}$ and $\{h_i\}$.

4. Portfolio selection problem

Let us consider the Markowitz mean-variance model [19] for the portfolio selection problem as stated below,

$$\text{Risk}: \quad \text{Min}\sum_{i=1}^{N}\sum_{j=1}^{N} x_i \sigma_{ij} x_j \quad (2)$$

$$\text{Mean Return}: \quad \text{Max}\sum_{i=1}^{N} \mu_i x_i \quad (3)$$

$$\text{Subject to} \quad \sum_{i=1}^{N} x_i = 1 \quad (4)$$

$$0 \le x_i \le 1, \quad i = 1, \ldots, N \quad (5)$$

where N is the number of different assets, μ_i is the mean return of asset i, and σ_{ij} is the covariance between returns of assets i and j. The decision variable x_i represents the fraction of capital to be invested in asset i. Eqs. (2) and (3) are two cost functions that should be solved with constraints (4) and (5). μ_i is the mean return of asset i in n intervals of time, i.e.

$$\mu_i = \sum_{t=1}^{n} \frac{W_{ei}(t) - W_{bi}(t)}{W_{bi}(t)},$$

where W_{bi} is the ith asset value at the beginning and W_{ei} is the ith asset value at the end of each interval.

A feasible solution of the portfolio selection problem is an optimal solution if there is no other feasible solution improving one objective without deteriorating the other. Usually, multiobjective optimization problems such as those in [20] have multiple non-dominated solutions. This set of solutions form an *efficient frontier*. For the problem defined in Eqs. (2)–(5), the efficient frontier is an increasing curve that gives the best tradeoff between mean return and risk (variance).

In this paper we change the multi-objective problem into a multimodal problem with a single objective function as follows:

$$\text{Minimize} \quad \lambda \cdot \left[\sum_{i=1}^{N}\sum_{j=1}^{N} x_i \sigma_{ij} x_j\right] + (1-\lambda) \cdot \left[-\sum_{i=1}^{N} \mu_i x_i\right] \quad (6)$$

$$\text{Subject to} \quad \sum_{i=1}^{N} x_i = 1 \quad (7)$$

$$0 \le x_i \le 1, \quad i = 1, \ldots, N \quad \text{and} \quad 0 \le \lambda \le 1 \quad (8)$$

Here $\lambda = 0.5$, for the equal effect of risk and mean return.

5. Solving portfolio selection problem using long range spin glass

To solve the portfolio selection problem, as studied in [6,29], each asset is supposed to be a spin having a value between 0 and 1. A glass (network) of such spins has an energy function as indicated in Eq. (1). To solve the portfolio selection problem in Eq. (6), the following relationship is observed for long range spin glass, i.e. $m = N$ (when $m < N$, the energy variation of the spin and its neighbors would not necessarily correspond to the global energy changes).

$$J_{ij} = -2\lambda\sigma_{ij} \quad (9)$$

$$h_i = (1-\lambda)\mu_i \quad (10)$$

Eqs. (9) and (10) refer to the *interaction* among spins and the *external field* energy of each spin, respectively. Search for optimal solutions begins with an initial assignment of each spin to $1/N$ (or randomly assigned). Then, any of the various search strategies can be used, in order to put the system in its minimum energy. At any moment (spin flip or spin change), a spin is randomly selected and ε is added to the spin's value ($\varepsilon = 0.05$, is a small fixed value). Then the values of neighboring spins change in such a way that they always meet constraints (7) and (8).

To maintain the first constraint, whenever ε is added to each spin's value ($x_i := x_i + \varepsilon$), the value ε/m is reduced from each of the spin's m neighbors ($x_j := x_j - (\varepsilon/m)$). This ensures that the sum of all spin values remains at 1. If $x_i \ge 1$, then $x_i := 1$ and its extra value is reduced from ε. Also if for a neighbor $x_j - (\varepsilon/m) \le 0$, then $x_j := 0$ and the difference is added to x_i. Considering the last two cases, the second constraint (Eq. (8)) is also maintained.

5.1. Using simulated annealing (SA)

Temperature and cooling schedule plays a central role in SA strategy [21]. The system's temperature is usually initialized to a high value to allow all possible states to be initially producible, i.e. more global exploration. The system is then gradually cooled to allow better local search. To do so, the temperature of the glass is considered to be initially set to $T_0 = 1$ (at high temperatures all states can occur). Each time the changes are applied, the temperature is decreased until it reaches near zero. Temperature variations is calculated as follows:

$$T(n) = \frac{T_0}{n^2}, \quad n \ge 1 \quad (11)$$

where n represents the number of epochs. The stop condition is the iteration of a single result in the number of defined steps continually with regard to defined precision. For example, all experiments' results have been measured by ten same results with a precision of 10^{-7}.

In this method, for any change in spin, the spin glass's energy is computed. If the energy is reduced, changes are accepted; otherwise they are accepted with a probability $e^{-(\Delta E/T)}$. This process is

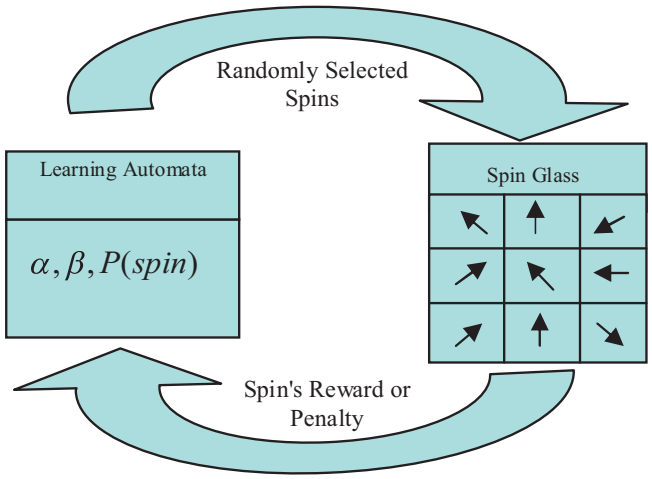

Fig. 2. Spin glass and learning automata interaction.

repeated until either the minimum energy is reached or the glass has fully cooled. The algorithm is as follows [6]:

Algorithm 1. SA Spin Glass [6]

Begin

1 Randomly place the possible assets into the spin glass
2 Initialize assignment of each spin to $\frac{1}{N}$
3 Select one spin randomly.
4 Change the state of selected spin i by ε (very small change) and change all the neighborhood spins for satisfying constraints (7) and (8).
5 Calculate the energy of the changed spin and its neighbor spins
$(m = N)\left(E_{new} = \sum_{i=1}^{m} E_i\right)$
6 $\Delta E = E_{new} - E_{old}$
7 If $\Delta E < 0$ then accept this change else
8 If $\Delta E > 0$ then accept this change with probability $e^{-(\Delta E/T)}$
9 Continue this process (go to line 3) with decreasing temperature till either ΔE remains near 0 for several iterations (i.e. the system has reached steady state, or T has reached a small enough value (system has cooled)

End

In the above algorithm, E_{old} and E_{new} are glass's energy before and after applying a change, and T is the system's temperature at the time of applying the change. This SA-based algorithm ensures convergence to global solutions if T is reduced sufficiently slowly. However, this also means a slow rate of convergence. In the below two algorithms, we investigate two alternative heuristics that *choose* the spin to be flipped based on a given criteria, hence aiming for faster convergence.

5.2. A hybrid with learning automata (LA-SA)

The first approach is based on learning automata. Learning automata here aims to reward spin selections that yield improved responses, hence leading to faster convergence. Learning automata is an abstract model which randomly selects an action from a finite set of actions and applies it onto the environment. The environment assesses the selected action and informs automata of the results by a reinforcement signal in the form of a reward or penalty [7]. Correspondingly, the automaton changes its internal condition and selects the next action. Fig. 2 shows the interaction between learning automata and the spin glass (as a random environment). The environment is shown by a triplet $V \equiv \{\alpha \quad \beta \quad P\}$ in which α is input set, β is output set, and P is a vector of spin selection probabilities. Here, β has two members (for reward) a and b (for penalty) and uses the following linear learning algorithm [27]:

$$\textit{Reward}: \quad p_i(n+1) = p_i(n) + a\left(\sum_{j=1}^{m} p_j(n)\right), \tag{12}$$

$$\textit{Penalty}: \quad p_i(n+1) = (1-b)\cdot p_i(n) \tag{13}$$

where j is the neighbor index, a is reward parameter, b is penalty parameter, and $p_i(n)$ is the probability of selecting spin i at step n. Whenever a spin's change improves the glass performance, that spin receives a reward and its neighborhood spins receive penalties (divided equally among neighbors, i.e. $a\left(\sum_{j=1}^{m} p_j(n)\right)/m$), so the probability of that spin's selection increases at next iteration. Likewise, whenever a spin's change deteriorates the glass performance, the spin receives penalty while its neighborhood spins receive rewards (divided equally among neighbors), and the probability of that spin's selection decreases. In the automata of the suggested method, the value of $a = b = 0.001$ is taken.

Portfolio selection problem is presented based on spin glasses and learning automata combined model such as Fig. 2 and Algorithm 2. Algorithm begins after initializing the spins and the value of selected spin changes at each step. If this value makes the system perform better, it receives a reward, but if it makes the performance worse, it receives a penalty. Spin's reward and penalty causes the probability of spin's selection to change. After several iterations, better spins have a better chance of being selected, so undesirable states are less likely to be explored, and the algorithm's rate of convergence increases.

Algorithm 2. LA-SA Spin Glass

Begin

1 Randomly place the possible assets into the spin glass
2 Initialize assignment of each spin to $\frac{1}{N}$
3 Select one spin randomly based on Learning Automata's selection probability
4 Change the state of selected spin i by ε (very small change) and change all the neighborhoods spins for satisfying constraints (7) and (8).
5 Calculate the energy of the changed spins and its neighbor spins
$(m = N)\left(E_{new} = \sum_{i=1}^{m} E_i\right)$
6 $\Delta E = E_{new} - E_{old}$
7 If $\Delta E < 0$ then accept this change and issue a reward to the spin, else
8 If $\Delta E > 0$ then accept this change with probability $e^{-(\Delta E/T)}$ and issue a reward to the spin.
9 Else, ignore changes and issue a penalty to the spin
10 Continue this process (go to line 3) with decreasing temperature till either ΔE remains near 0 for several iterations (i.e. the system has reached steady state, or T has reached a small enough value (system has cooled)

End

5.3. A hybrid with extremal optimization (EO-SA)

EO is a local-search heuristic algorithm that was presented by Boettcher [8]. In EO, every decision variable in the current individual is considered as a spin and always change worse one to improve the overall fitness. It specifically causes to improve spins' state and moves toward the optimal solution very fast. Since, EO operates locally only on one individual, at each step improves one spin that it may be trapped in local optimum and also fluctuate around it. Therefore, to escape from local or guiding to the global solution, SA is a good candidate to compose by EO.

In contrast to the above LA-SA method, EO-SA gives the highest selection probability to a spin that has lowest local energy (from Eq. (14)) hence avoiding locally optimal solutions. Here, spins are ordered based on their local energy. At each step, a '*superior*' spin

is selected based on its given probability in Eq. (16), with higher probability given to lower energy spins. After several iterations, because the glass moves toward lower energy and each spin affects its neighboring spins, many spins have lower energy than their initial values, i.e. the given value in Eq. (15) reduces. Hence, the system's correlation increases, and the change in each spin leads to a change in many other spins which leads to *self-organized critically* (SOC) [7,9]. In this state, any small change leads to major changes in the system, so it is expected that most possible states are accessible. Therefore, one can easily escape local optimal solutions and survey most possible states for the system.

Algorithm 3 describes how the EO-SA method can be applied to spin glasses. In this algorithm, φ_i is local energy of each spin in Eq. (14). Spin glasses' total energy can then be obtained from a sum of φ_is in Eq. (15).

$$\varphi_i = x_i \left(\frac{1}{2} \sum_{j=1}^{m} J_{ij} \cdot x_j + h_i \right) \tag{14}$$

$$E(\{x_i\}) = -\sum_{i=1}^{N} \varphi_i \tag{15}$$

All φ_is are computed and ordered in increasing order at each step and selected based on the power law distribution in Eq. (16). The selected spin's value is then changed. If this change leads to lower (better) glass energy, it is accepted; otherwise, it is accepted with a probability of $e^{-(\Delta E/T)}$.

Algorithm 3. EO-SA Spin Glass

Begin
1 Initialize with local behavior.
2 Calculate φ_i for each spin and sort them with an increasing order.
3 Selected spins with power law distribution in Eq. (16) based on calculated φ_i.
4 Change the state of the selected spin i by ε (very small change) and change all the neighboring spins to satisfy Eqs. (7) and (8).
5 Calculate the energy of the changed spin and its neighbor spins $(m = N)\left(E_{new} = \sum_{i=1}^{m} E_i \right)$
6 $\Delta E = E_{new} - E_{old}$
7 If $\Delta E < 0$ then accept this change, else
8 If $\Delta E > 0$ then accept this change with probability $e^{-(\Delta E/T)}$
9 Continue this process with decreasing temperature till either ΔE remains near 0 for several iterations (i.e. the system has reached steady state, or T has reached a small enough value (system has cooled)
End

The selection of every spin at each step depends on the following Eq. (2):

$$k = (1 + (N^{1-\tau} - 1) \cdot \alpha)^{1/1-\tau} \tag{16}$$

where k refers to the selected spin's number, whose set is ordered from low energy to high energy spins, and $0 \le \alpha \le 1$ is a random number. When $\tau = 0$, the algorithm acts like SA at high temperature; and when $\tau \to \infty$, the algorithm always selects the spin with minimum energy. Therefore it can be expected that the above algorithm has the power law distribution equaling $P_k \propto k^{-\tau}$ in which $1 \le k \le N$ [8].

6. Experimental results

In order to verify the effectiveness of the above algorithms, the benchmarked "standard efficient frontier" (Pareto Front) is compared with the efficient frontier resulting from the proposed methods.

Experiments on the benchmark data were originally performed in [22]. These data are obtained from five major stock exchange markets, during the time period extending from March 1992 to September 1997. These five stock exchange markets include Hang Seng in Hong Kong (31 assets), Deutscher Aktien Index (DAX 100) in Germany (85 assets), Financial Times London Stock Exchange (FTSE100) in Britain (89 assets), Standard & Poor's (S&P 100) in USA (98 assets), and Nikkei in Japan (225 assets). These benchmark data are presented in text file format as follows: *number of assets* (N); and for each asset i ($i = 1, \ldots, N$): *mean return* as well as *standard deviation of return*; for all possible pairs of assets: i, j, *correlation* between asset i and asset j.

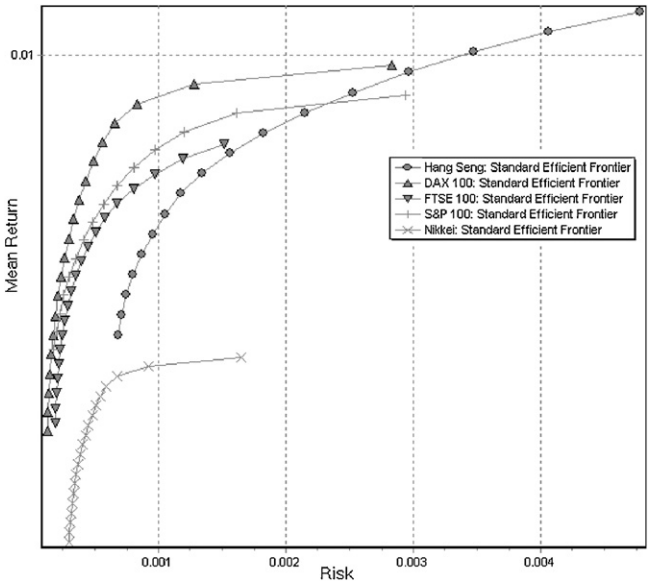

Fig. 3. Efficient frontier for benchmark data from five major stock markets as reported in [22].

The efficient frontier for each of these five stock markets in the available time period is characterized by mean return as in Eq. (2) and variance of return as in Eq. (1). Fig. 3 illustrates this efficient frontier for the benchmark data.

Three sets of tests are performed to analyze the spin glass behavior as follows. Firstly, spin glass's accuracy and rate of convergence is compared for the proposed two approaches, i.e. LA-SA and EO-SA, as well as the more conventional SA approach. Secondly, the resulting efficient frontier is compared with the benchmark's efficient frontier. Thirdly, the reliability of presented algorithms and phase transition analysis are tested and compared with those of SA.

All of the experiments were performed using Borland Delphi 6.0 running on a Pentium 2.0 GHz PC, under Windows XP operating system. It should be mentioned that each epoch equals 50 spin flips.

6.1. Comparing SA, EO-SA and LA-SA

As seen in Fig. 4, all three spin glasses begin under similar random initial states and reach same final states using the three SA, EO-SA and LA-SA algorithms for the S&P stock market. However, they have significantly different rates of convergence. Based on the results seen in all studied stock markets, EO-SA method quickly approaches the final ground states, but fluctuates around the final states for many iterations before reaching it. Because the number of qualified spins increases at each iteration, there is increasing correlation among spins. Therefore, changes in each spin lead to changing in many other spins and causes fluctuating around the response range. In comparison, LA-SA method is slower in reaching near the final ground state; but once there, it converges to a value near the ground state quickly and does not fluctuate. This is because it first tries to create a proper spin selection distribution

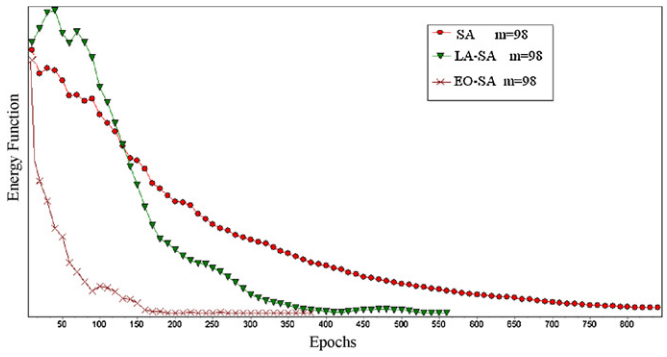

Fig. 4. Comparing SA, EO-SA and LA-SA algorithms' rate of convergence for S&P stock market.

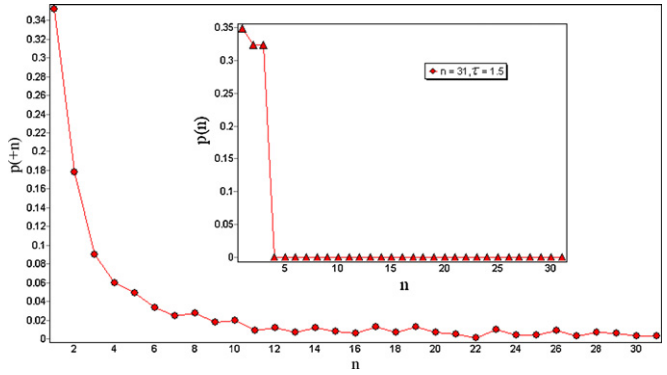

Fig. 5. Comparing the distribution of spins' selection resulting from EO-SA (external chart) and LA-SA (internal chart), after reaching final state for Hang Seng stock market. n is the size of the portfolio.

before converging to a local optimum about the ground state. In contrast, the SA method has a simple random behavior and slowly moves to the ground state.

To compare EO-SA and LA-SA, some other considerations should also be taken. For example, Fig. 5 plots the final spin selection probability vs. spins that are ordered in descending selection probability levels. As this figure illustrates, experiments show that while these two methods are different, they can reach similar results in the neighborhood of glass's ground states. This conclusion is illustrated in an experiment carried out for Hang Seng stock market, with $\tau = 1.5$ for EO-SA and $a = b = 0.001$ for LA-SA. As Fig. 6 illustrates, both distributions of spins' selection reach the same general distribution/ranking of spin selection. This state occurs when the glass has converged.

Other experiments indicate that there can be a correspondence between values of τ in EO-SA and values of a (reward) and b (penalty) in LA-SA where both algorithms often reach similar states during latter stages of convergence. It can also be noted that many of the poorer performing assets have zero probability of selection in LA-SA, i.e. inferior spins have no chance of being selected (Fig. 5, internal chart). This is usually seen for problems that have many local optima around the global optimum and can lead to the entrapment of the algorithm in local optima. This problem is not limited to this example, but also in other automata based problems [7]. This is the differentiating point between automata and extremal methods, for all spins *always* have the probability of being selected in extremal method and the probability is never zero. Therefore, the accuracy of extremal method using spin glass is more than learning automata for addressing this problem using spin glass.

Table 1 shows a comparison between the computation time and the accuracy of reaching ground states in the three mentioned methods. While all three methods reach the ground state and have a generally comparable accuracy, EO-SA method has a faster convergence than the other two methods in our experiments. The only exception is in problems with fewer than 50 assets. It seems that EO is computationally more demanding because of the required sorting of spins, hence the convergence time for fewer than 50 assets is more than LA-SA method's. But EO-SA method is better than two other methods in other stock markets, for example in Nikkei stock market, EO-SA method responds better than SA and LA-SA methods and its response is sharper and more exact. In other words, the improved search heuristic outperforms the added computational overhead of EO's sorting.

7. Reliability test

Test of reliability is performed by running the algorithm n times independently with the same data [10]. To pass the test, the test

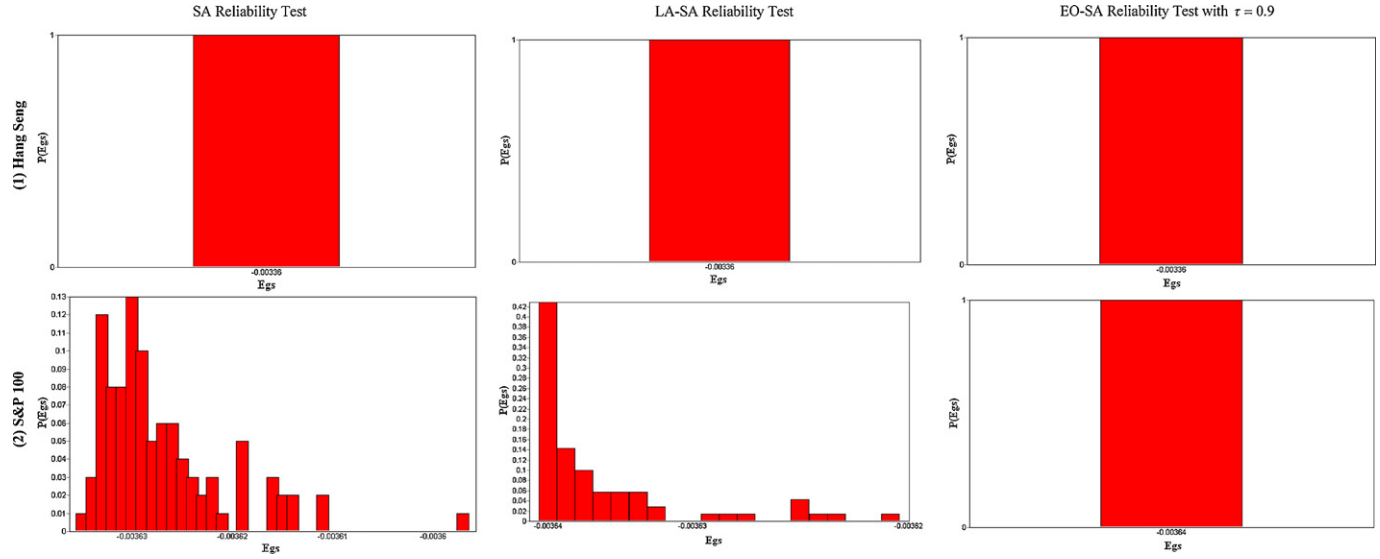

Fig. 6. The reliability test of Hang Seng and S&P 100 stock market (where Egs is glass energy at the ground state). Reliability test from 100 trials indicates that the algorithm's final value has a small variation over actual minimal value. $P(E_{gs})$ shows the probability of finding true ground state (E_{gs}). As indicated, In S&P 100, EO-SA shows the smaller variation than LA-SA and SA and hence EO-SA is more reliable than LA-SA and SA. Due to brevity, the results of the other three benchmarks are summarized in Table 1.

Table 1
Comparing the three algorithms (SA, LA-SA and EO-SA) for accuracy and rate of convergence over 100 independent runs for each of the five World's stock markets. Each approach is evaluated in terms of convergence time (shown in ms), average minimum energy during the runs, and accuracy defined by the ratio of obtained minimum energy over actual minimal energy.

Stock market	SA			LA-SA			EO-SA		
	Convergence time (ms)	Average minimum energy	Accuracy (%)	Convergence time (ms)	Average minimum energy	Accuracy (%)	Convergence time (ms)	Average minimum energy	Accuracy (%)
Hang Seng (31 assets)	2140	−0.00336	99.20	833	−0.00337	99.59	1253	−0.00337	99.40
DAX (85 assets)	19,111	−0.00412	99.10	6064	−0.00411	98.40	4091	−0.00412	99.18
FSTE (89 assets)	25,040	−0.00335	99.20	7741	−0.00335	99.20	6169	−0.00335	99.24
S&P (98 assets)	22,828	−0.00363	99.36	10,597	−0.00363	98.84	4048	−0.00363	99.3
Nikkei (225 assets)	214,045	−0.00142	98.91	109,761	−0.00139	97.91	82,103	−0.00142	98.89

runs are expected to yield similar results with small variation. To do so, the reliability test of the three algorithms is carried out for the five benchmarks. For brevity, the analysis of Hang Seng and S&P stock markets is shown here. Results are shown in the form of frequency chart in Fig. 6. Due to the numerous figures with multiple λ, only one example is provided for $\lambda = 0.5$ and the others are similar.

It is done in such a way that spin glass's minimum energy (E_{gs}) in the ground state is counted and the probability to reach that state is also shown ($P(E_{gs})$). The average and accuracy of the final energy states is given in Table 1. Experimental results from 100 trials indicate that the algorithm's final value has a small variation over the actual minimal value and hence algorithm has a good accuracy. In other words, final spin glass's energy at each trial is in the range of best responses. Even though the movement toward this final response is random in the above algorithms, they consistently reach the ground state. For example, as indicated in Fig. 6, in S&P 100, EO-SA shows the smaller variation than LA-SA and SA and hence EO-SA is more reliable than LA-SA and SA.

8. Optimization frontier

The final Pareto front from EO-SA algorithm is shown in Fig. 7 (the results are similar for LA-SA). This figure shows the validity of energy reduction and avoiding local optima for the five mentioned stock markets. The standard frontier for different λs is also drawn. For having a Pareto frontier, λ is considered in the range of 0.05–0.95 with 0.05 increments. For any λ, the spin glass's optimization state is found and its risk and capital return values are illustrated with points. The validity of the presented algorithm in finding a diverse set of solutions with different λ is seen by comparing the resulting and standard (benchmark) Pareto fronts. Since

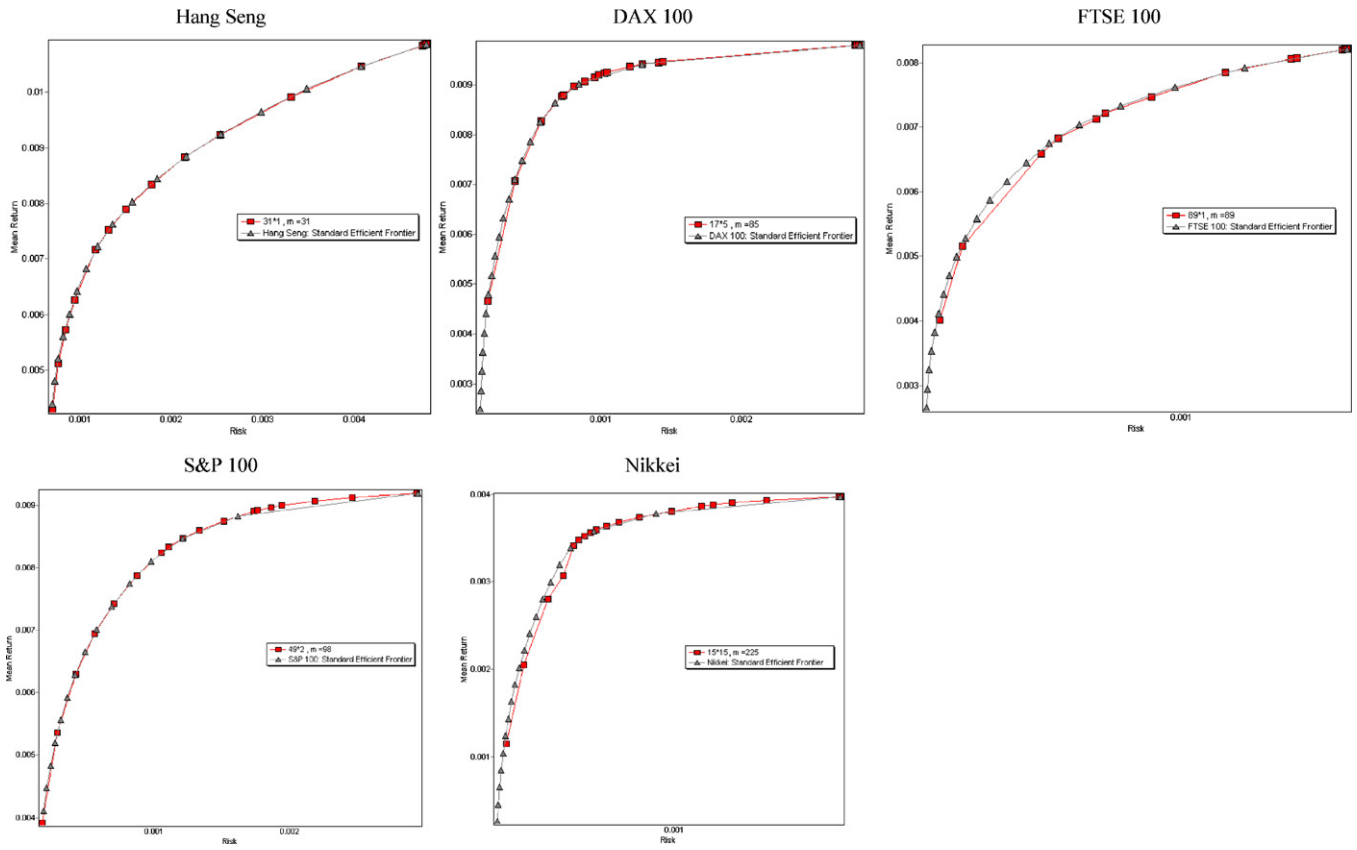

Fig. 7. Efficient frontier obtained from EO-SA algorithm compared to standard efficient frontier from benchmark data [22].

Table 2
Comparison between SA, LA-SA, EO-SA, GA, TS and NN. The ground state of glass for each stock is calculated 100 times and the average of convergence time (CT) and average mean return error (AMRE) are listed. The experiment results mentioned for GA, TS, and NN are extracted from paper [16].

Stock		SA	LA-SA	EO-SA	GA	TS	NN
Hang Seng ($N=31$)	AMRE ($\times 10^3$)	1.1203	0.81	1.16	1.1321	1.1237	1.2316
	CT (ms)	2140	833	1253	5110	19,432	39,021
DAX 100 ($N=85$)	AMRE ($\times 10^3$)	2.3896	0.9	1.1211	2.4457	2.6668	1.5776
	CT (ms)	19,111	6064	4091	16,320	45,764	106,921
FTSE ($N=89$)	AMRE ($\times 10^3$)	0.9512	0.9303	0.8132	0.7310	0.7357	1.2513
	CT (ms)	25,040	7741	6169	16,590	57,100	110,632
S&P ($N=98$)	AMRE ($\times 10^3$)	1.7251	1.975	0.7321	1.3236	1.3130	1.7922
	CT (ms)	22,828	10,597	4048	18,797	50,987	121,121
Nikkei ($N=225$)	AMRE ($\times 10^3$)	0.5458	1.787	0.9148	1.1415	0.5510	1.4737
	CT (ms)	214,045	109,761	82,103	579,876	590,831	2,793,091

the surface of the overall optimization frontier is generally covered, one can draw that the presented method gives response for any λ.

9. Phase transition analysis

The temperature at phase transition is defined as the temperature at which the likelihood of reaching the glass's actual minimum state suddenly decreases [11].

Fig. 8 illustrates the spin glass temperature at phase transition. As can be observed, before phase transition the probability of reaching minimum state (E_{gs}/E_{min}) nears 1 (E_{gs} is the ground state energy of glass and E_{min} is the actual minimum of cost function). As the temperature increases, this probability is expected to gradually decrease, but this decrease does not occur till reaching near the temperature at phase transition, at which time there is a sudden change in glass behavior. Here, phase transition is defined to occur where the likelihood ratio E_{gs}/E_{min} is decreased by 1%.

As illustrated in Fig. 8, most of the benchmarks reach phase transition at 1.12×10^{-6} temperature (as indicated by a vertical line) for the SA algorithm. This phase transition occurs at a higher temperature for LA-SA in Fig. 9, i.e. at 6.3×10^{-6} the glass has consistently reached ground state. In other words, since temperature is lowered gradually in these algorithms, LA-SA can be said to find minimum state sooner than SA. Furthermore, as Fig. 10 suggests, most of the benchmarks with EO-SA find their minimum states even at higher temperatures (7.9×10^{-6}) as compared with LA-SA and SA, prompting EO-SA as the algorithm that converges soonest.

The above phase transition analysis also confirms the conclusions of Table 1. Specifically, this experiment shows that, in EO-SA, the temperature of phase transition is higher than SA and LA-SA, and accordingly better rate of convergence.

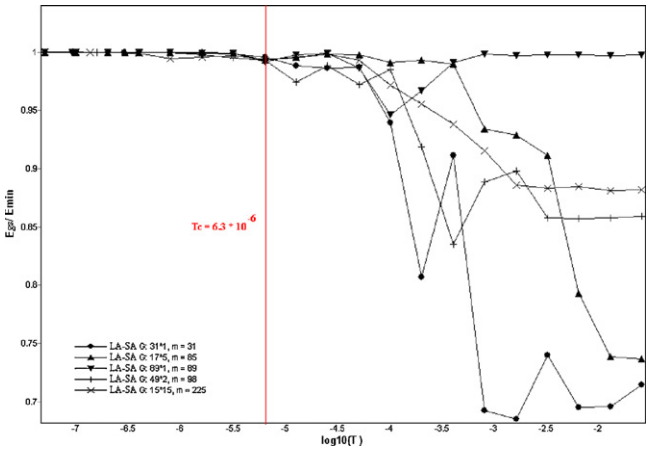

Fig. 9. Portfolio selection phase transition phenomena based on LA-SA for the five benchmark stock market data; transition temperature is approximately 6.3×10^{-6}.

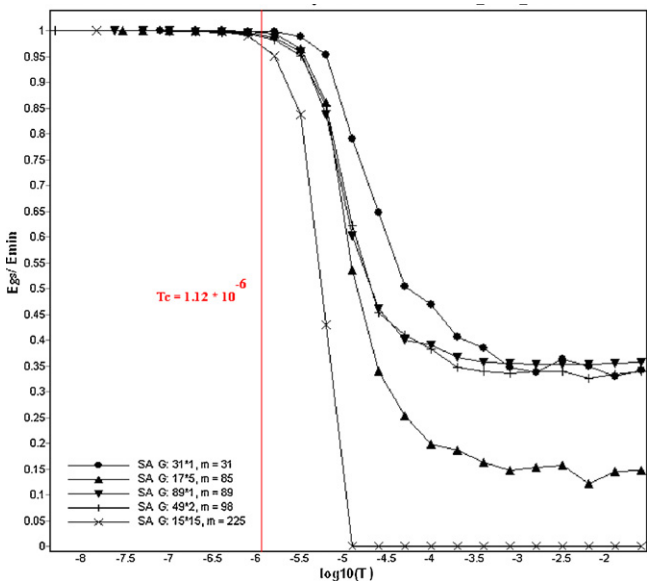

Fig. 8. Portfolio selection phase transition phenomena based on SA for the five benchmark stock market data; transition temperature is approximately 1.12×10^{-6}.

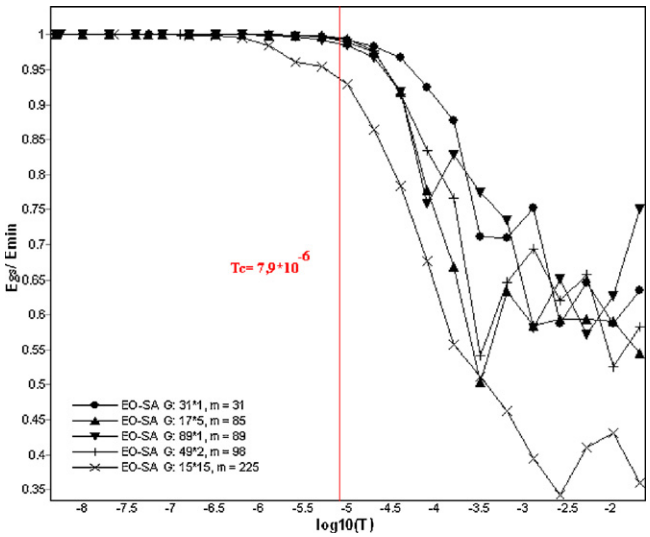

Fig. 10. Portfolio selection phase transition phenomena based on EO-SA for the five benchmark stock market data; transition temperature is approximately 7.9×10^{-6}.

10. Comparison with other heuristics

Now let us take a look at some numerical results in Table 2. This table is complement of Table 1 in paper [16] that shows the comparison of Neural Network (NN), Genetic Algorithm (GA), Tabu Search (TS) and SA.

Table 2 allows some kind of comparison between SA, LA-SA and EO-SA results and those of paper [16]. In Table 2, average mean return error (AMRE) of the mentioned methods and their convergence time (CS) are shown. (The convergence time of the proposed method and the current methods in paper [16], due to the differences in computers and similar simulation software, is not the same. Because, the authors implement the GA, TS and NN, according to the author's understanding of the paper [16]; in Table 2 the new CT results have been listed. Some differences in the experimental run time results are observed but the conclusion seems generally correct.) AMRE shows the degree of closeness of measurements of an objective function to its actual value. With regard to the computation times, EO-SA is the most efficient algorithm followed by LA-SA, SA, TS, GA and finally NN. As compared EO-SA with other heuristic methods, EO-SA improves considerably (except for the first stock market Hang Seng) AMRE and convergence time for all different stock markets.

11. Conclusion

In this paper, two hybrid approaches are proposed for finding long range spin glass's ground state based on learning automata and extremal optimization. These two hybrid approaches aim to address the slow convergence of the more conventional SA.

In the first method, a hybrid LA-SA selects the *best* spin at each flip, and if the spin yields a better solution, the likeliness of its selection is increased at the next flip. This process continues until the probability distribution in spins' selection appears. Once the glass reaches this distribution, the glass quickly converges to its ground state. The rate of convergence in this method is therefore higher than SAs as indicated by the test experiments. What is important here is that in the resulting distribution, the selection of less qualified spins gradually becomes zero, i.e. they have little/no chance of being selected in the future and hence increasing the probability of entrapment in local optima. This is one of the barriers in using learning automata.

In contrast, the EO-SA method selects spins with minimum energy with a higher probability. A flip (change) in any spin leads to changes in its neighboring spins. If all of these changes reduce spin glass's energy, more and more spins will be better qualified and the correlation between spins increases. A process of self organizing criticality then occurs where the change in each spin leads to changes in many spins, allowing the glass to escape local optima more easily. As the experiments on phase transitions illustrate, the temperature at phase transition is elevated, hence the rate of convergence is improved.

A comparison of experiments shows the superiority of EO-SA to SA and LA-SA, particularly in larger glasses. LA-SA only has a faster rate of convergence for smaller glasses. Results also indicate that while the two methods are different, they change the structure of spin glass in such a way that they reach similar behaviors at the latter stages near convergence as well as similar results. This is also confirmed by phase transition analysis and reliability test.

Finally, to reach an objective assessment, it should be mentioned that performance of these two algorithms depends a great deal on the value of their parameter settings τ for EO-SA and a and b for LA-SA. At the end, performance of the algorithm is compared to that of three famous heuristic algorithms (Genetic Algorithm, Tabu Search and Neural Network) in solving optimal portfolio selection problem. In this experiment, EO-SA improves considerably AMRE and convergence time for all different stock markets.

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